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IES | GATE | PSU

STRUCTURAL ANALYSIS

For

CIVIL ENGINEERING

STRUCTURAL ANALYSIS

SYLLABUS

Structural Analysis: Statically determinate and indeterminate structures by force/ energy methods; Method of superposition; Analysis of trusses, arches, beams, cables and frames; Displacement methods: Slope deflection and moment distribution methods; Influence lines; Stiffness and flexibility methods of structural analysis.

ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	2	-	12
2004	3	-	9
2005	2	-	10
2006	-	3	6
2007	1	4	9
2008	-	3	6
2009	-	1	2
2010	1	1	3
2011	-	-	-
2012	-	-	-
2013	1	3	7
2014 Set-1	2	2	6
2014 Set-2	1	1	3
2015 Set-1	1	1	3
2015 Set-2	1	1	3
2016 Set-1	1	1	3
2016 Set-2	1	1	3
2017	1	1	3

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1 DETERMINACY / INDETERMINACY & STABILITY OF STRUCTURES

1.1 Introduction

The aim of structural analysis is to find forces/moments in various components of structures. If these forces can be found out by the use of equation of static equilibrium, the structure is called statically determinate.

1.2 Equation of Static Equilibrium

- In a 2-D structure or planer structure (in which all members and forces are in plane only) the equations of equilibrium are

$$\left.\begin{array}{l}\sum F_X = 0 \\ \sum F_Y = 0 \\ \sum M = 0\end{array}\right\} \text{ 3 nos.}$$

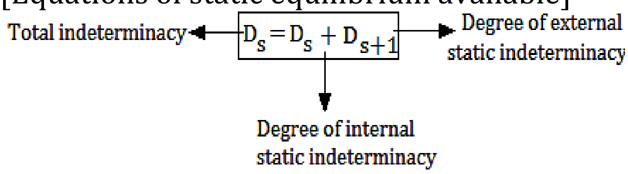
- In a. 3-D structures or space structures (in which members and forces are in 3rD). The equations of equilibrium are

$$\left.\begin{array}{ll}\sum F_X = 0 & \sum M_X = 0 \\ \sum F_Y = 0 & \sum M_Y = 0 \\ \sum F_Z = 0 & \sum M_Z = 0\end{array}\right\} \text{ 6 nos.}$$

- If however, member forces cannot be found by equations of static equilibrium alone, the structure is called statically indeterminate.
  In this case additional equations needed are obtained by relating the applied loads & reactions to the displacements or slopes known at different points on the structure. These equations are called Compatibility equations.

1.3 Degree of Static Indeterminacy (D_S)

D_S = [Number of unknown forces in members or at support reactions]- [Equations of static equilibrium available]



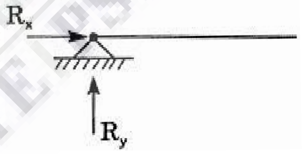
Thus total indeterminacy of a structure in excess of external indeterminacy is called internal indeterminacy.

1.4 Support Reactions

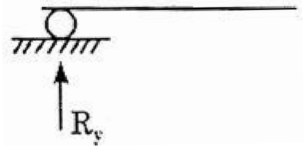
Restraining of deformations at support gives rise to support reactions.

Plane Structure

Fixed support restrains A_x, A_y, 0_Z, hence support reactions are R_x, R_y and M_z (3 nos.)

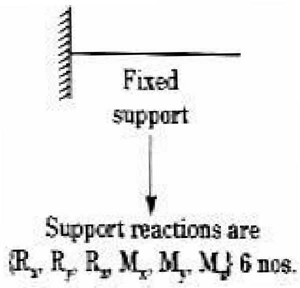


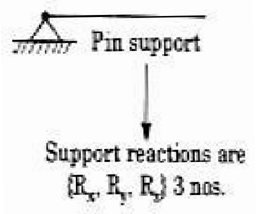
Pin support restrains A_x, A_y. Hence support reactions are R_x and R_y (2 nos.)

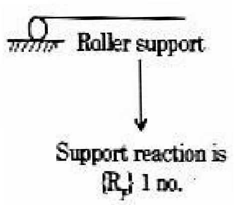


Roller support restrains A_y. Hence support reaction is R_y.

1.5 Space Structure







1.6 External Indeterminacy (D_{SE})

D_{SE} = [Total No. support reactions in the structure]-[No. Equations of static equilibrium available]

$\Rightarrow D_{Se} = R - 3 \rightarrow$ for plane structure

$= R - 6 \rightarrow$ for space structure

1.7 Internal Indeterminacy (D_{SI})

$D_{Se} = D_S - D_{Se} =$ Total indeterminacy – External indeterminacy

1.8 Degree of Static Indeterminacy for Frames

Frames are rigid jointed structures. All the joints are made rigid by providing extra restraint R'. The structure is then cut to make it, Open Tree like determinate structure.

$D_{Se} = 3C - R' \rightarrow$ for plane frame

$= 6C - R' \rightarrow$ for space frame

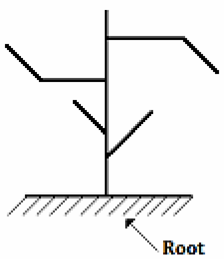
C = No. of cuts to make structure determinate

R' = no. of restraints applied to make all joints rigid.

1.9 Open Tree like Structure

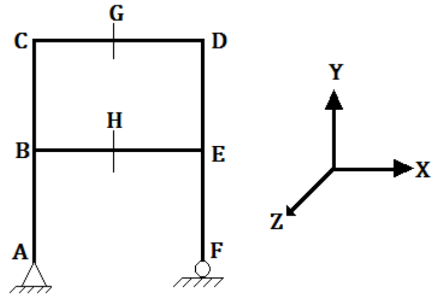
The structure is cut in such a way that each individual cut part looks like a tree as shown here. Note that

1. Tree should have only one root.
2. Tree cannot have a closed looped branch



1.10 Justification for the Formula

To prove D_S = 3C - R', we take plane frame as shown below.



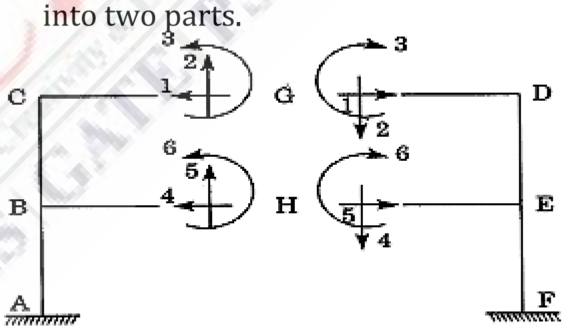
No. of restraint that is required to be applied to make the structure rigid is

1. no. at A i.e. moment M_{AZ}
2. nos. at F i.e. R_X and M_{FZ}

   Thus, Total no. of added restrain = R'= 1 + 2 = 3

   This R' = 3 corresponds to 3 known reaction conditions i.e. M_{Az} = 0, M_{FZ} = 0, R_X = 0.

   When the structure is cut it gets divided into two parts.



Open Tree Like Structure

- If these 6 reactions at the cut section are known, the structure becomes completely determinate i.e. forces in all members AB, BC, DE, EF, CD, BE can be determined.

  Thus, no. of unknowns are: (6 reactions at the cut section—3 known conditions i.e. R_X = M_{AZ} = M_{FZ} = 0)

  As the_no of cuts are C = 2, the no. of unknowns can be written as 3 x 2 — 3 i.e. 3 x C — R'

$\Rightarrow$ $\boxed{\text{Degree of static indeterminacy} = 3C - R'}$

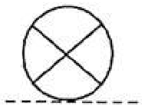
In 3-D frame, at any cut section no. of reactions are 6 $[R_X, R_Y, R_Z, M_X, M_Y, M_Z]$.

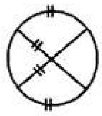
Hence,

Degree of static indeterminacy for 3D frame

$= 6C - R'$

Note:
The above structure is externally determinate, because the number of support reactions is three and equation of static equilibriums is also 3. But the structure is internally indeterminate to 3rd degree.





4-cuts required

When the structure is not having a support as in figure shown the degree of static indeterminacy = 3C - R - 3
Where
C = No of cuts req.
R' = No. of restrain added
3= No. of equation of static equation
Hence
$D_S = 3 \times 4 - 0 - 3 = 9$

1.11 Restraining Support (For Plane and Space Frame)

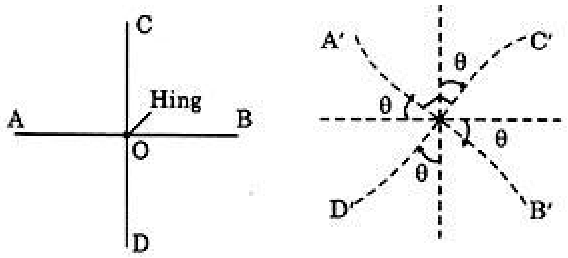
No. restrained required to make a joint rigid
= No. of support reactions for fixed support – No. of reactions at actual support

1.12 Restraining Member/Joint

Plane Frame (Joint having hinge)
No. of restraining moments required at a joint where m-members meet = (m - 1).
Space Frame [Joints having hinge]
No. of restraining moment required at a joint where m—member meet = 3(m — 1).

1.13 Justification for Applying (m-1) Restraining Moment at Joint Having Hinge in 2D Frame.

If the joint O had been rigid, rotation of one member with respect to other will be zero as shown in figure below.



1.14 Another Method [For Rigid Frame]

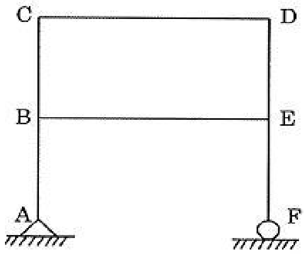
In plane frame, every member carries three forces (BM, SF, axial force).
Hence, Total no. of unknowns = 3m + r
Where,
m = no. of members and
r = no. of support reactions.
At each joint, no. of equations. of equilibrium available
$= \sum F_X = 0, F_Y = 0, M_Z = 0$
Total no. of eq. of equilibrium=3J, J = no. of joints
Hence, degree of static indeterminacy
$D_S = 3m + r - 3J$

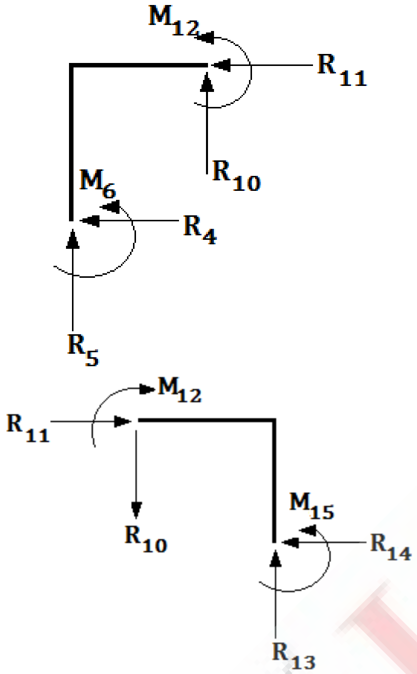
However if the frame carries hinges, then D_S is reduce further by Z(m' - 1), where m' = no. of members meeting at the hinge. Summation term has components, one for each hinge
$D_S = 3m + r - 3J -$
$\sum(m'-1)$
Similarly for space frame
$D_S = m + r - 1$
Due to internal hinge in space frame,
$D_S = 6m + r - 1 - 6j - \sum(m'-1)$

1.15 Justification

Let us take a frame like as shown below.
1. m = no. of members = 6
   They are AB, BC, CD, DE, EF, BE.
2. No. of support reactions = R = 3 nos.
   They are R_{yA}, R_{xA} and R_{yF}

3. J = no. of joints = 6





They are A, B, C, D, E, F

$\Rightarrow D_S = 3m + r - 3J$

$= 3 \times 6 + 3 - 3 \times 6 = 3$

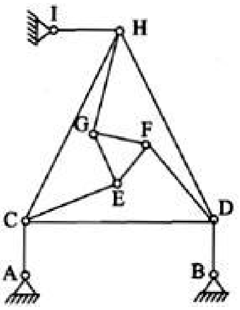
If we make free body diagram of the structure, it will look like this: (Note for each Joints one free body diagram has beam made)

GATE QUESTIONS

Topics **Page No**

1 DETERMINACY & INDETERMINACY

Q.1 The following two statements are made with reference to the planar truss shown below:



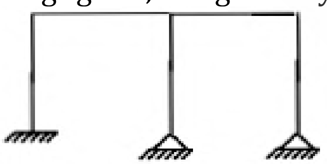
I. The truss is statically determinate
II. The truss is kinematically determinate

With reference to the above statements, which of the following applies?

a)Both statements are true
b)Both statements are false
c)II is true but I false
d)I is true but II is false

[GATE- 2000]

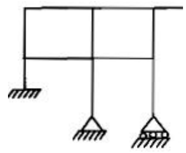
Q.2 The degree of static indeterminacy, Ns and the degree of kinematic indeterminacy, N_k for the plane frame shown below, assuming axial deformations to be negligible, are given by:



a) N_s = 6 and Nk = 11
b) N_s = 6 and Nk = 6
c) N_s = 4 and Nk = 6
d) N_s = 4 and Nk = 4

[GATE-2001]

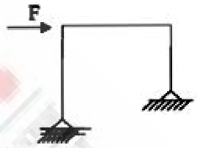
Q.3 For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of static indeterminacy, d, and the degree of kinematic indeterminacy, k, are



a)d = 3 and k = 10
b)d = 3 and k = 13
c)d = 9 and k = 10
d)d = 9 and k =13

[GATE – 2004]

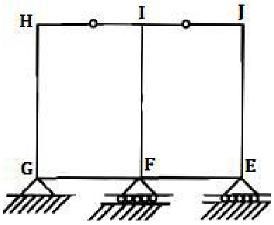
Q.4 Considering beam as axially rigid, the degree of freedom of a plane frame shown below is



a) 9 b) 8
c) 7 d) 6

[GATE – 2005]

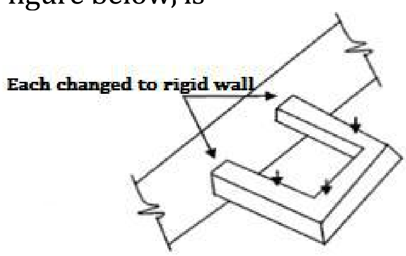
Q.5 The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure below, is



a) 8 b)7
c) 6 d) 5

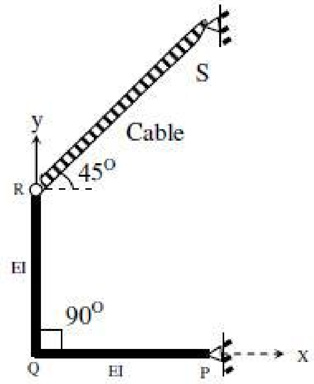
[GATE-2008]

Q.6 The degree of static indeterminacy of a rigidly jointed frame in a horizontal plane and subjected to vertical load only, as shown in figure below, is



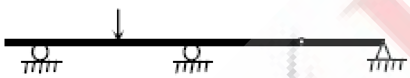
a) 6 b)4
c) 3 d) 1
[GATE-2009]

Q.7 The degree of static indeterminacy of a rigid jointed frame PQR supported as shown in the figure is



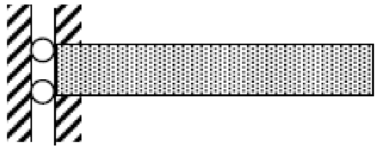
a) Zero b) One
c) Two d) Unstable
[GATE-2014]

8. The static indeterminacy of the two-span continuous beam with an internal hinge, shown below, is _____



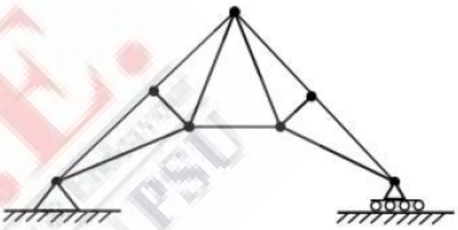
[GATE-2014]

9. A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotational) with stiffness k_x, k_y and k_θ respectively. The limiting values of k_x, k_y and k_θ are



a) ∞, 0, ∞ b) ∞, ∞, ∞
c) 0, ∞, ∞ d) ∞, ∞, 0
[GATE-2015]

Q.10 The kinematic indeterminacy of the plane truss shown in the figure is



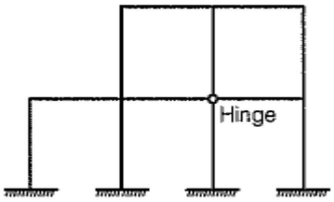
a) 11 b) 8
c) 3 d) 0
[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
(d)	(c)	(d)	(b)	(d)	(a)	(a)	0	(a)	(a)

ASSIGNMENT QUESTIONS

Q.1 Total degree of indeterminacy (both internal and external) of the plane frame shown in figure is



a) 10 b) 12
c) 12 d) 15

Q.2 If there are m unknown member forces, r unknown reaction components and j number of joins, then the degree of static indeterminacy of a pin-joined plane frame is given by
a) m + r + 2j b) m – r + 2j
c) m + r – 2j d) m + r – 3j

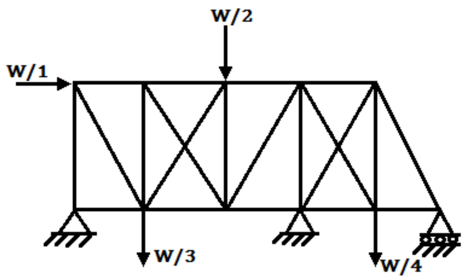
Q.3 Degree of kinematic indeterminacy of a pin joined plane frame is given by
a) 2j - r b) j – 2r
c) 3j – r d) None of these

Q.4 A pin-joined plane frame is unstable if
a) (m + r) < 2j b) m + r = 2j
c) (m + r) > 2j d) None of these

Q.5 The degree of static indeterminacy of a pin-joined space frame is given by
a) m + r – 2j b) m + r – 3j
c) 3m + r – 3j d) m + r + 3j

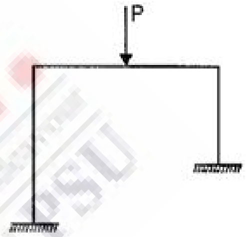
Q.6 The degree of kinematic indeterminacy of a pin-joined space frame is given by
a) 2j – r b) 3j - r
c) j – 3r d) j – 3r

Q.7 The degree of static indeterminacy of the pin-joined plane frame shown in figure is



a) 1 b) 2
c) 3 d) 4

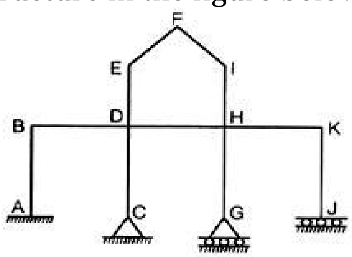
Q.8 The portal frame as shown in the given frame is statically indeterminate to the



a) first degree b) second degree
c) third degree d) None of these

Q.9 A perfect plane frame having n number of members and j number of joints should satisfy the relation
a) n < 2j - 3 b) n = 2j – 3
c) n > 2j - 3 d) n = 3 – 2j

Q.10 Neglecting axial deformation, the kinematic indeterminacy of the structure in the figure below is



a) 12 b) 14
c) 20 d) 22

Q.11 Consider the following statements:
1) The displacement method is more useful when degree of kinematic indeterminacy is greater than the degree of static

indeterminacy greater than the degree of static indeterminacy

2) The displacement method is more useful when degree of kinematic indeterminacy is less than the degree of static indeterminacy. Indeterminacy is less than the degree of static indeterminacy

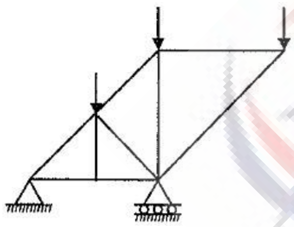
3) The force method is more useful when degree of static indeterminacy is greater than the degree of kinematic indeterminacy.

4) The force method is more useful when degree of static indeterminacy is less than the degree of kinematic indeterminacy.

Which of the statements are correct?

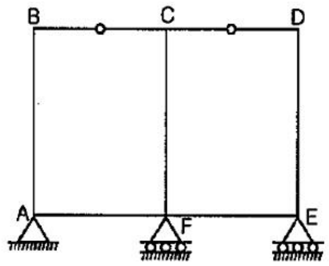
a) 1 and 3 b) 2 and 3
c) 1 and 4 d) 2 and 4

Q.12 The pin-joined frame shown in the figure is



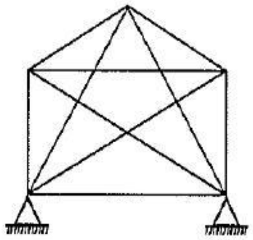
a) a perfect image
b) a redundant frame
c) a deficient frame
d) None of the above

Q.13 The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure is



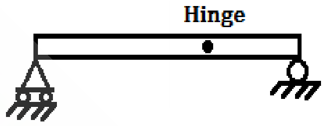
a) 8 b) 7
c) 6 d) 5

Q.14 What is the degree of static indeterminacy of the plane structure as shown in the figure below?



a) 3 b) 4
c) 5 d) 6

Q.15 The prismatic is show in the figure below.



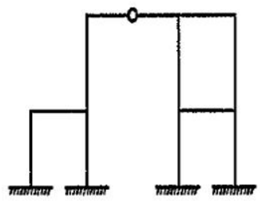
Consider the following statements:

1) The structure is unstable.
2) The bending moment is zero at supports and internal hinge.
3) It is a mechanism.
4) It is statically indeterminate.

Which of these statements are correct?

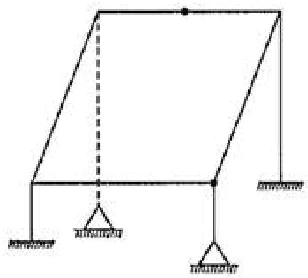
a) 1, 2, 3 and 4 b) 1, 2 and 3
c) 1 and 2 d) 3 and 4

Q.16 What is the statical indeterminacy for the frame shown below?



a) 12 b) 15
c) 11 d 14

Q.17 The statical indeterminacy for the given 3D frame is



a) 8 b) 6
c) 9 d) 12

EXPLANATIONS

Q.1 (c)

The degree of indeterminacy

$D_s = r_e + (30 - r_r) - 3(j + j')$

Number of external reactions r_e = 3 + 3 + 3 + 3 = 12

Number of rigid joints,
J = 10

Number of joints at which releases are located,
j' = 1

Number of members,
m = 12

As the hinge is located at a point where 4 members meet. Hence it is equivalent to three hinges. Therefore number of releases r_r = 3.

$\therefore\ D_s = 12 + (3\times12 - 3) - 3(10+1)$

$= 12 + 33 - 33 = 12$

Q.2 (c)

Q.3 (a)

Q.4 (a)

Q.5 (b)

Q.6 (b)

Q.7 (d)

$D_{se} = r_e - 3$

$r_E = 2 + 2 + 1 = 5$

$\therefore\ D_{se} = r_e - 3 = 5 - 3 = 2$

Internal indeterminacy

$D_{si} = m - (2j - 3)$

No of members, m = 21
Number of joints, j = 11

$\therefore\ D_s = 21 - (2\times11 - 3)$

$= 21 - 19 = 2$

$\therefore\ D_s = D_{se} + D_{si} = 2 + 2 = 4$

Q.8 (c)

$D_{se} = r_e - 3 = 6 - 3 = 3$

$D_{si} = 3C = 3\times0 = 0$

$D_s = D_{se} + D_{si} = 3 + 0 = 3$

Q.9 (b)

A perfect plane frame means a determinate structure, so

$n - (2j - 3) = 0$

$\therefore\ n = 2j - 3$

Q.10 (b)

Q.11 (d)

Force method is useful when

$D_s < D_K$

Displacement method is useful when $D_k < D_s$

Q.12 (a)

Degree of indeterminacy

$n = (m + r_E) - 2j$

$= (9 + 3) - 2\times6 = 0$

Since the degree of indeterminacy is zero and the frame is stable so it is a perfect frame.

Q.13 (d)

$D_s = D_{se} + D_{si}$

$D_{se} = r_E - 3$

$= 4 - 3 = 1$

$D_{si} = 3C - r_R = 3C - \sum(m^1 - 1)$

$= 3\times2 - (2-1) + (2-1)$

$= 6 - 2 = 4$

$\therefore\ D_s = D_{se} + D_{si} = 5$

Q.14 (b)

For plane truss degree of indeterminacy