I.C.E.™

Inspiring Creativity & Endeavour

IES | GATE | PSU

SIGNALS AND SYSTEMS

For

ELECTRICAL ENGINEERING

INSTRUMENTATION ENGINEERING

ELECTRONICS & COMMUNICATION ENGINEERING

SIGNALS AND SYSTEMS

SYLLABUS

ELECTRONICS AND COMMUNICATION ENGINEERING

Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

ELECTRICAL ENGINEERING

Representation of continuous and discrete-time signals; shifting and scaling operations; linear, time-invariant and causal systems; Fourier series representation of continuous periodic signals; sampling theorem; Fourier, Laplace and Z transforms.

INSTRUMENTATION ENGINEERING

Periodic and aperiodic signals. Impulse response, transfer function and frequency response of first- and second order systems. Convolution, correlation and characteristics of linear time invariant systems. Discrete time system, impulse and frequency response. Pulse transfer function. IIR and FIR filters.

ANALYSIS OF GATE PAPERS

	ELECTRONICS			ELECTRICAL			INSTRUMENTATION		
Exam Year	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total
2003	4	3	10	-	-	-	3	9	21
2004	3	6	15	1	2	5	3	8	19
2005	6	6	18	-	4	8	3	3	9
2006	3	3	9	3	4	11	-	8	16
2007	1	4	9	2	5	12	2	3	8
2008	2	8	18	2	7	16	3	7	17
2009	3	5	13	1	3	7	2	4	10
2010	2	3	8	2	4	10	6	3	12
2011	3	4	11	2	2	6	7	3	13
2012	2	3	8	2	4	10	4	3	10
2013	7	3	13	6	1	8	7	3	13
2014 Set-1	4	4	12	2	3	8	4	2	8
2014 Set-2	3	3	9	2	2	6	-	-	-
2014 Set-3	4	4	12	3	3	9	-	-	-
2014 Set-4	4	4	12	-	-	-	-	-	-
2015 Set-1	2	3	8	2	2	6	2	6	14
2015 Set-2	5	4	13	1	4	9	-	-	-
2015 Set-3	4	6	16	-	-	-	-	-	-
2016 Set-1	4	4	12	4	3	10	4	4	12
2016 Set-2	1	2	5	5	2	9	-	-	-
2016 Set-3	3	4	11	-	-	-	-	-	-
2017 Set-1	3	4	11	2	2	6	2	3	8
2017 Set-2	2	3	8	2	2	6	-	-	-

CONTENTS

1 INTRODUCTION TO SIGNALS & SYSTEMS

1.1 INTRODUCTION TO SIGNAL

Signals play a major role in our life. In general, a signal can be a function of time, distance, position, temperature, pressure, etc., and it represents some variable of interest associated with a system. For example, in an electrical system the associated signals are electric current and voltage. In a mechanical system, the associated signals may be force, speed, torque etc. In addition to these, some examples of signals that we encounter in our daily life are speech, music, picture and video signals.

"A signal is a function representing a physical quantity which conveyed some amount of Information."

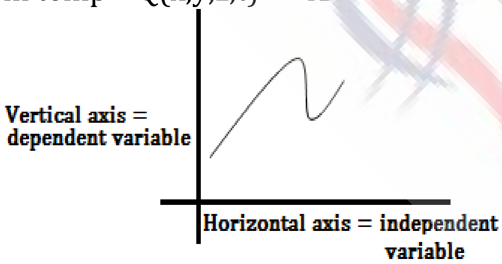
Signals can very more than one independent variables.

Example Speech (passing through a telephone line), one variable → A cosωt

Image – I(x,y) (2D)

T.V. Picture – I(x,y,t) → 3D

Room temp – Q(x,y,z,t) → 4D



1.2 CLASSIFICATION OF SIGNALS

1.2.1 CONTINUOUS-TIME AND DISCRETE - TIME SIGNALS

Signals can be classified based on their nature and characteristics in the time domain. They are broadly classified as (i) continuous–time signals and(ii) discrete-time signals. A continuous-time signal is a mathematically continuous function and the function is defined continuously in the time domain. On the other hand, a discrete-time signal is specified only at certain time instants. The amplitude of the discrete-time signal between two time instants is not defined.

A. Continuous time and Discrete time Signals:- (Change in Horizontal Axis)

A signal x(t) is a continuous –time signal if it is a continuous variable. If it is a discrete variable than it is defined as discrete time signal.

Notations:

x(t) → continuous time signal

x[n] → discrete time signal

B. Discrete-time Signals – Sequences

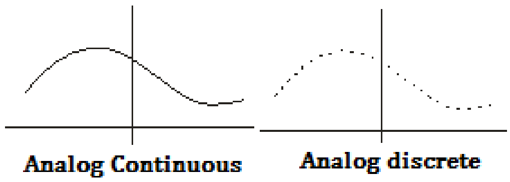
A discrete–time signal has a value defined only at discrete points in time and a discrete-time system operates on and produces discrete-time signals. A discrete-time signal is a sequence is a sequence which is a function defined on the positive and negative integers, that is, x(n) = {x(n)} = {.....x(-1), x(0), x(1), ...} where the up-arrow ↑ represents the sample at n = 0.

If a continuous-time signal x(t) is sampled every T seconds, a sequence x(nT)results. In the sample interval, T forconvenience, the sample interval T is taken as 1 second and hence x(n) represents the sequence.

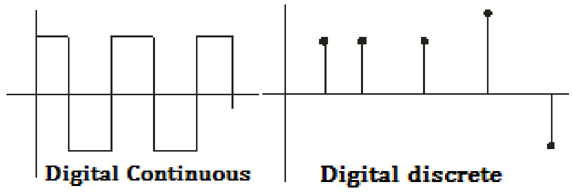
1.2.2 ANALOG AND DIGITAL SIGNALS: (Change in Vertical Axis)

1) Analog:- A Signal can take on infinite number of distinct values in amplitude.

2) Digital:- A Signal can take on finite number of distinct values in amplitude.

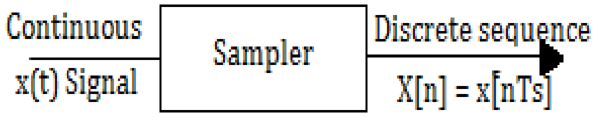


3) Digital Continuous



1. Analog Continuous Signal
2. Analog Discrete Signal
3. Digital Continuous Signal
4. Digital Discrete Signal

Sampler : It is a device (switch) which convert continuous time signal to discrete time sequence.



$T_S \rightarrow$ Sampling time

$t = nT_S \quad n = 0, \pm 1, \pm 2, \pm 3$

$T_S \rightarrow$ Sampling time

Example: $x(t) = e^{-2t}\, u(t)$ in discrete form

Solution: $x(nT_S) = e^{-2nTS}\ [nT_S]$, for T_S=1 sec.

$x[n] = e^{-2n}\, u[n]$

→ Sampling – discretizing – X – axis

→ Quantization–discretizing–Y – axis

Example :-For given the continuous time signal

$$x(t) = \begin{cases} 1-|t| & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

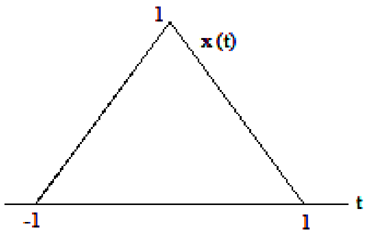
Determine the discrete time sequence obtained by uniform sampling of x(t) with a sampling interval of

a) 0.25 sec.

b) 0.5 sec.

c) 1.0 sec.

Solution:- Draw x(t)



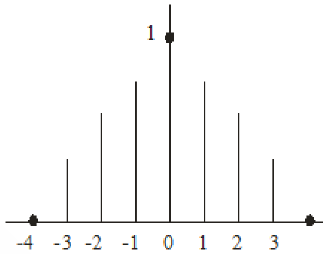
$$x(t) = \begin{cases} 1-t & 0 \le t \le 1 \\ 1+t & 0 \ge t \ge -1 \end{cases}$$

a) T_S = .25sec., $t = n.T_S$

n = 0,1,2,3,4 …………………

x[n] = x[n/4]

x[n] = {0,.25, 05, .75, 1, .75, .5, .25, 0 ……………..}
↑

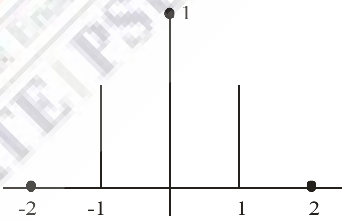


b) T_S = .5sec. $t = nT_S$

n = 0,1,2,3,4 …………………

$x_2[n] = x[n/2]$

x[n] = [.0, 0.5, 1, .5, 0]
↑

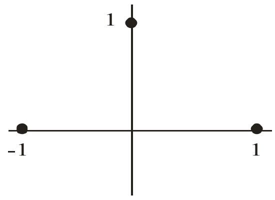


c) T_S =1sec., $t = nT_S$

n = 0,1,2,3,4

$x_3[n] = x[n]$

x[n] = {………..0, 1, 0………..} = s[n]
↑



T_S ↑ no. of samples in given interval deceases f_S ↓, T_S ↑, or f_S ↑, T_S↓

1.2.3 REAL AND COMPLEX SIGNAL

A signal x(t) is a real signal if its value is a real number, if x(t) complex number, then it complex Signal.

$x(t) = x_1(t) + jx_2(t) \rightarrow$ imaginary part

Real part

$j = i = \sqrt{-1}$

$x_1(t) \rightarrow$ real part = $x_r(t)$

$x_2(t) \rightarrow$ imaginary part = $x_i(t)$

$$< x(t) = \tan^{-1}\left(\frac{x_2(t)}{x_1(t)}\right) \quad \begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix}$$

Two form to representational

1) Cartesian form

2) Polar form

(1) Cartesian form :- $x + jy$,

Example :

a) $e^{-j\pi/2}$

b) $e^{-j5\pi/2}$

c) $\sqrt{2}e^{-j\pi/4}$,

d) $\sqrt{2}e^{-j9\pi/4}$

Solution :

a) –j b) +j

c) $\sqrt{2}\left[\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right] = 1 + j$ d) 1 –j

(2) Polar Form :- $re^{j\theta}$

Example :

a) 5

b) -2 -3j

c) j(1 –j)

d) $\frac{1+j}{1-j}$

e) $\left[\frac{\left(\sqrt{2} + j\sqrt{2}\right)}{\left(1 + j\sqrt{3}\right)}\right], \ -\pi \le \theta \le \pi$

Solution :

a) $5e^{j\theta}$ b) $\sqrt{13}e^{-j\tan^{-1}\left(\frac{3}{2}\right)}$

c) $e^{-j\pi/4}.\sqrt{2}$ d) $e^{j\pi/2}$

e) $e^{-j\pi/12}$

1.2.4 DETERMINISTIC AND RANDOM SIGNALS

Deterministic and Non-deterministic Signals

Deterministic signals are functions that are completely specified in time. The nature and amplitude of such a signal at any time can be predicted. The pattern of the signal is regular and can be characterized mathematically examples of deterministic signals are

(i) $x(t) = \alpha t$ This is a ramp whose amplitude increases linearly with time and slope is α.

(ii) $x(t) = A \sin \omega t$. The amplitude of this signal varies sinusoidally with time and its maximum amplitude is A.

(iii) $x(n) = \begin{Bmatrix} n \ge 0 \\ 0 \quad \text{otherwise} \end{Bmatrix}$ is a discrete-time signal whose amplitude is 1 for the sampling instants $n \ge 0$ and for all other samples, the amplitude is zero.

A non-deterministic signal is one whose occurrence is random in nature and its pattern is quite irregular. A typical example of a non deterministic signal is thermal noise in an electrical circuit.

Deterministic: It's values are completely specific for any given time. (equation or formula).

Random Signal: - It take random values at any given time.

1.3 OPERATION ON A SIGNAL

(A) Shifting

(B) Scaling

(C) Inversion / Folding or reflection

A) Time Shifting :- $x(t - t_0)$

x(t) shifted in time by t_0

$t_0 > 0$ Right shift (delay)

$t_0 < 0$ Left shift (Advance)

(B) Time Scaling x(at)

x(t) is time scaled by a

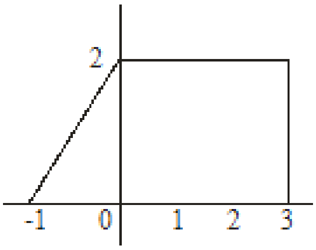
$|a| > 1$, It is multiplication or compression

$|a| < 1$, It is division or expression

(C) Time inversion x(-t)

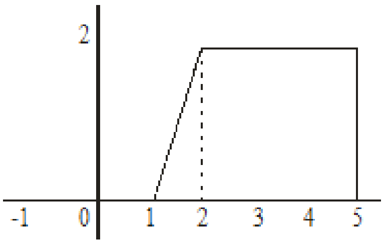
Folded about y – axis

Example : Let signal x(t) $-1 \le t \le 3$

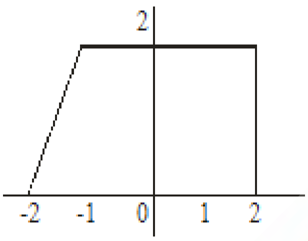


Solution :

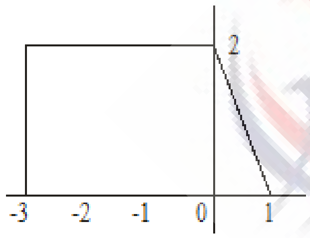
a) x(t) $+1 \le t \le 5$, delay



b) x(t+1) $-2 \le t \le 2$, Advance

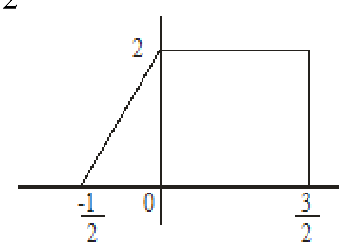


c) x(-t) $-3 \le t \le 1$, Inversion

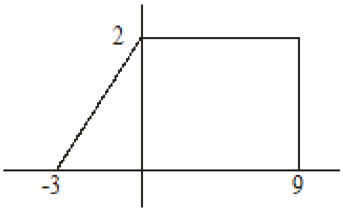


d) x(2t) $-1 \le t \le 3$

$$-\frac{1}{2} \le t \le \frac{3}{2}$$

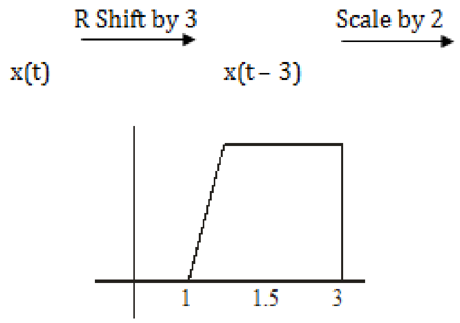


e) x(t/3) $-3 \le t \le 9$

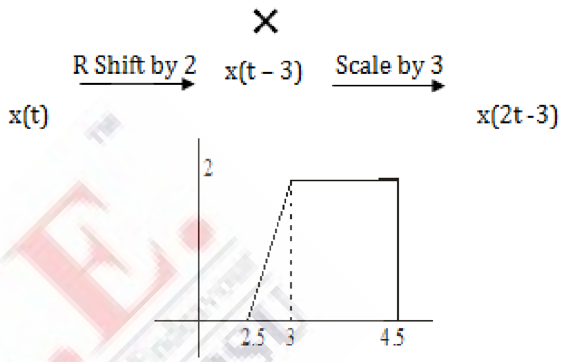


f) x(2t – 3)

Case – 1



Case - 2



Scaling can't be done before shifting

$-1 \le 2t - 3 < 3$

$2 \le 2t < 6$

$1 \le t \le 3$

Combined Operations

x(at – b) This can be realized in two possible sequences of operation:

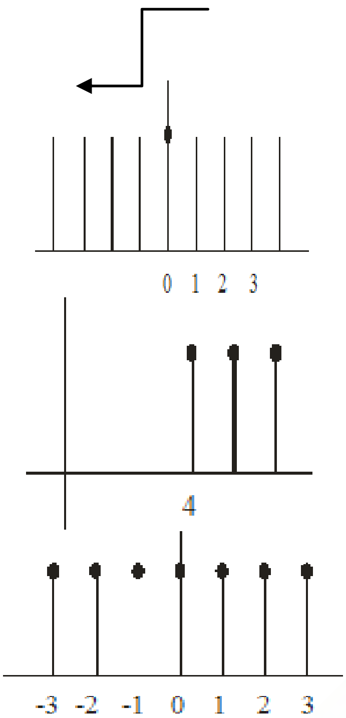
1. **Time** – shift x(t) by b to obtain x(t – b). Now time – scale the shifted signal x(t – b) by a (i.e., replace t with at) to obtain x(at – b).

2. **Time** – scale x(t) by a to obtain x(at). Now time-shift x(at) by b/a (i.e., replace t with t – (b/a) to obtain x[a(t – b/a)] = x(at – b). In either case, if a is negative, time scaling involves time reversal.

For example, the signal x(2t -6) can be obtained in two ways. We can delay x(t) by 6 to obtain x(t-t), and then time – compress this signal by factor 2(replace t with 2t) to obtain x(2t – 6). Alternately, we can first time-compress x(t) by factor 2 to obtain x(2t), then delay this signal by 3(replace t with t – 3) to obtain x(2t – 6).

Example : Given x[n] = 1- $\sum$ [n−1−k] k=3
Such that x[n] – u [mn – n_0] find m & n_0.

Solution : $x[n]=1-\sum_{k=3}^{\infty}\delta(n-1-k)$



u[-n] to get x[n] shift it by 3 unit
x[n] = u[-(n -3)]
x [n] = u[-n + 3]
Company with x[n] = u[mn – n_0]
m = -1, n_0 = -3

Example:-Given a sequence x[n], to generate y[n]= x[3 - 4n], which of following procedures would be correct?

Solution: → Let x(n) → $N_1 \le x(n) \le N_2$
x[3 - 4n] ↔ $N_1 \le x(n) \le N_2$
↔ $N_1 \le x(n) \le N_2$
↔ $N_1 \le 3 - 4n \le N_2$
↔ $N_1 - 3 \le -4n \le N_2 - 3$
↔ $\frac{3-N_1}{4} \ge n \ge \frac{3-N_2}{4}$

A) First delay x(n) by 3 Samples to generate $Z_1(n)$
$N_1 \le x_1(n) \le N_1 \rightarrow Z_1(n) - N_1 \le n-3 \le N_2$
$\rightarrow Z_1(n) - N_1 + 3 \le n \le N_2 + 3$

$Z_2(n)$ ↔ pick every 4^{th} sample of $Z_1(n)$
m = $1/4$, down sampling
$\frac{N_1+3}{4} \le n \le \frac{N_2+3}{4}$ $Z_2(n)$
$\rightarrow N_1 + 3 \le 4n \le N_2 + 3$
$x(3-4) \ne y(n)$ $y(n) = Z_2(-n)$
Time inverse
$y(n) \rightarrow -\frac{N_1+3}{4} \ge n \ge -\frac{N_2+3}{4}$

B) First advance x(n) by 3 Sample to generate Z_1 (n)
$N_1 \le x(n) \le N_2$ – $N_1 \le n+3 \le N_2$
– $N_1 - 3 \le n \le N_2$-3
$Z_2(n)$ ↔ pick every 4^{th} sample of Z_1 (n)
m = 1/4 down sampling
$Z_2 \rightarrow \frac{N_1-3}{4} \le n \le \frac{N_2-3}{4}$
$y(n) \leftrightarrow Z_2(-n)$
$y(n) = \frac{-N_1+3}{4} \ge n \ge \frac{-N_2+3}{4}$
$x(3-4n) = y(n)$

C) x(n) ↔ $N_1 \le x(n) \le N_2$
(Pick every 4^{th} sample of sequence)
$V_1(n) \rightarrow N_1 \le 4n \le N_2$
$V_1(n) \rightarrow \frac{N_1}{4} \le n \le \frac{N_2}{4}$
$V_2(n)$ = V1(-n) → time reverse
$V_2(n) \rightarrow \frac{N_1}{4} \le -n \le \frac{N_2}{4} \rightarrow -\frac{N_1}{4} \ge n \ge -\frac{N_2}{4}$
y(n) = V_2 (N + 3), V_2 time advance 3 unit to obtain y (n)
$V_2(n) \rightarrow \frac{N_1}{4} \ge n+3 \ge \frac{-N_2}{4}$
$\rightarrow -\frac{N_1}{4} - 3 \ge n \ge -\frac{-N_2}{4} - 3$
y(n) ≠ x(3 -4n)

D) First pick every forth sample of x(n) sequence
$V_1(n) \rightarrow N_1 \le 4n \le N_2$,
$V_1(n) \rightarrow \frac{N_1}{4} \le n \le \frac{N_2}{4}$

$V_2(n) = V_1(-n) - \text{time reverse}$

$V_2(n) \rightarrow \frac{N_1}{4} \leq -n \leq \frac{N_2}{4}$

$\rightarrow V_2(n) \rightarrow \frac{-N_1}{4} \geq n \geq \frac{-N_2}{4}$

y(n) = V_2(n-3), V_2 time delay 3 unit to obtain y(n)

$y(n) = -\frac{N_1}{4} \geq n - 3 \geq -\frac{N_2}{4}$

$\rightarrow y(n) = -\frac{N_1}{4} + 3 \geq n \geq -\frac{N_2}{4} + 3$

y(n) ≠ x [3 – 4n]

Example: x[n] = [-3, 5, 4, -2, 3] then write x[3n], x[2n]

Solution: Decimation or down sampled discard (n – 1) samples w.r.t. to origin

$x[3n] = [5, 3], \text{ discard } 2 \text{ sample}$
$\uparrow$

$x[2n] = [5, -2], \text{ discard } 1 \text{ sample}$
$\uparrow$

$x(n) \xrightarrow{x[n/2]} Z(n) \xrightarrow{Z[2n]} x(n)$

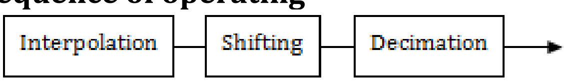
Decimation is the inverse of interpolation but interpolation is not inverse of decimation.

$x(n) \xrightarrow{x[2n]} Z(n) \xrightarrow{Z[n/2]} x(n)$ not equal

if x(n -2) → Signal is R shift by 2

if x(n-1, 3) $\leftrightarrow x\left(\frac{10n-13}{10}\right)$

Sequence of operating



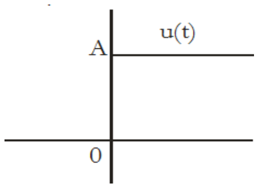
Interpolation should be proceed by decimation.

1.4 IMPORTANT SIGNALS

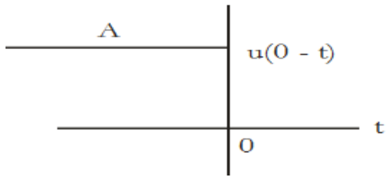
1.4.1

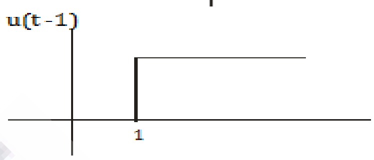
(1) Step Signal :-

$$Au(t) = \begin{cases} A & t > 0 \\ 0 & t > 0 \end{cases}$$

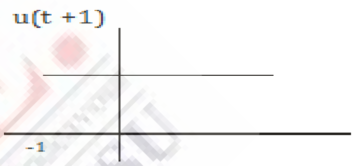


$$Au(t) = \begin{cases} A & t < 0 \\ 0 & t > 0 \end{cases}$$

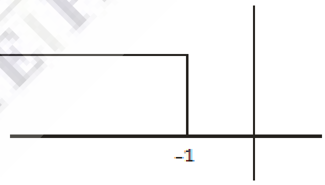




delay by 1 unit
u(t-1)

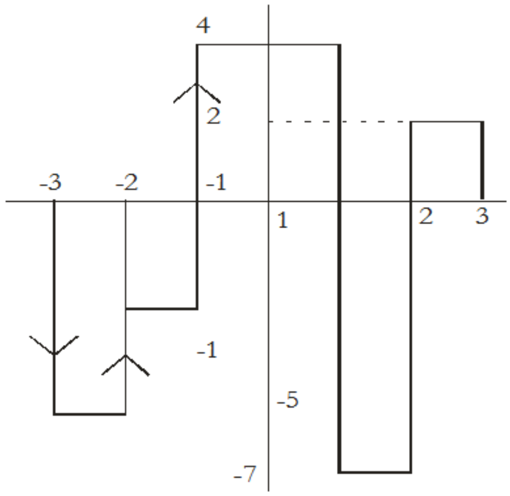


advance by 1 unit



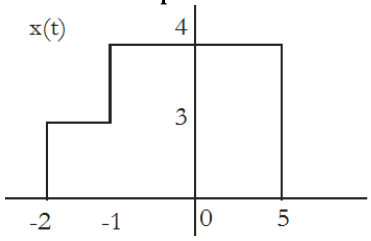
0 < - t – 1 <∞
1 < - t <∞
-1 > t > 1- ∞

Example : Write expression



$x(t) = -5u(t+3) + 4u(t+2)$
$+5u(t+1) - 11u(t-1)$
$+9u(t-2) - 2u(t-3)$

Example : Write expression

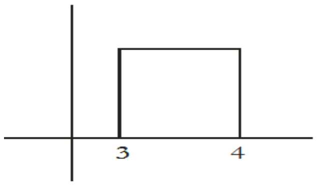


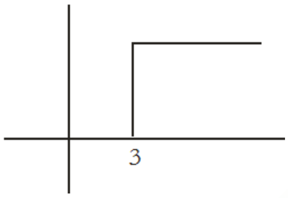
$$x(t) = 3u(t+2) + u(t+1) - 4u(t-5)$$

Example: Possible expression in terms of steps:

Solution :

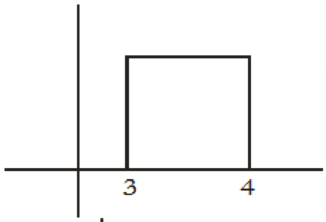
a)

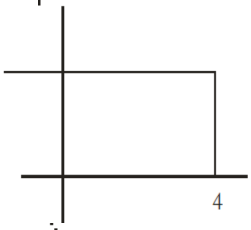


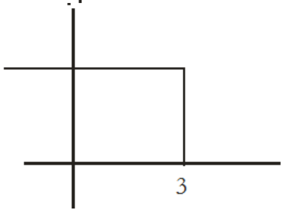


a) u(t-3) – u(t-4)

b)

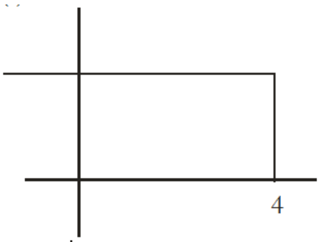


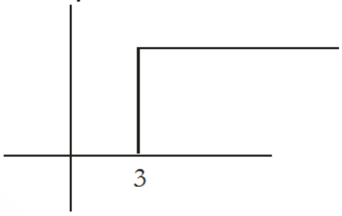




b) u(- t + 4) – u(- t + 3)

c)





c) $u(t - 3) \times u(t + 4)$

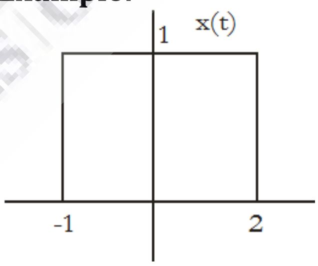
Relation between unit step signal and impulse signal

$$\delta(t) = \frac{d}{dt}u(t)$$

$$u(t) = \int_{-\infty}^{\infty} \delta(t)dt$$

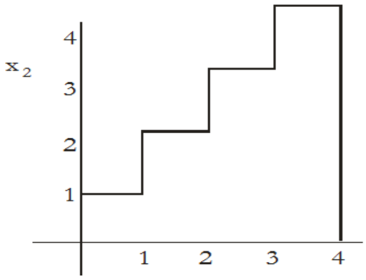
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Example:



Solution: $[u(t+1) - u(t-2)]$

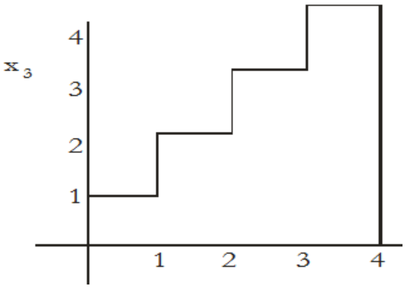
Example:



Solution :

$$[u(t-1) + u(t) + u(t-2) + u(t-3) - 4u(t-4)]$$

Example:

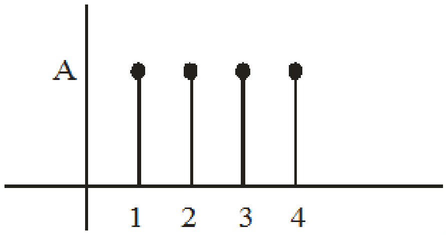


Solution :

$4u(t-1)-u(t-1)-u(t-2)-u(t-3)-u(t-4)$

1.4.2 Discrete Step Signal

$$Au[n]=\begin{cases}A & \text{at } n\geq 0\\ 0 & \text{at } n<0\end{cases}$$



Example: Sketch the following

(1) 3u(n + 4)

(2) u(n – 6)

(3) 3u (6 – 2n)

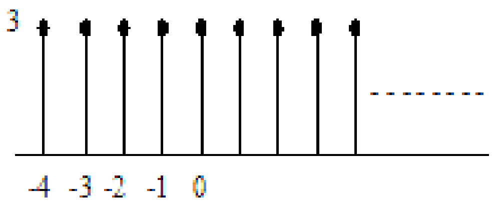
(4) u(n) – u(n – 1) = $\delta(n)$

First difference of step function known as impulse response

= S(n) – S(n – 1)

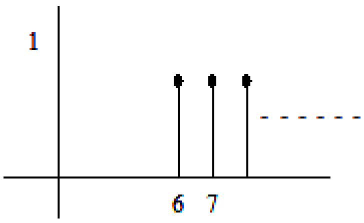
$u(n) = \sum_{k=-\infty}^{\infty}\delta(k)$

$x[n] = \sum_{k=-\infty}^{\infty}x(k)\delta(n-k)$



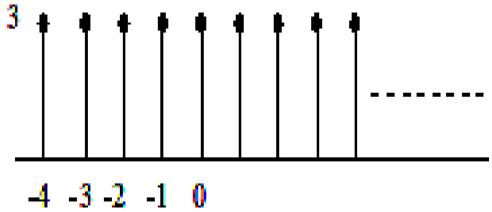
$0 \leq n + 4 < \infty$

$-4 \leq n \leq \infty$



$0 \leq n - 6 < \infty$

$6 \leq n \leq \infty$



$0 \leq 8 - 2n < \infty$

$-8 \leq -2n \leq \infty$

$-4 \leq -n \leq \infty$

1.4.3 Impulse Function

Unit-impulse Function

Indicate that the area of the impulse function is unity and this area is confined to an infinitesimal interval on the t-axis and concentrated at t = 0. The unit impulse function is very useful in continuous-time system analysis. It is system characteristics. In discrete-time domain, the unit-impulse signal is called a unit-sample signal.

Unit impulse function or direct delta function.

$$\delta(t)=\begin{cases}0, & t\neq 0\\ \infty, & t=0\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)dt=1 \text{ or } \int_{-\epsilon}^{\epsilon}\delta(t)dt=1$$

Properties of impulse signal

1) Scaling : $\delta(at)=\frac{1}{|a|}\delta(t)$

or δ (a(t – t_0) = $\frac{1}{|a|}\delta$ (t – t_0)

if a = -1

$\delta(-t)=\delta(t)$ even function

2) $\int_{-\infty}^{\infty}\delta(t-t_0)dt=\int_{-\infty}^{\infty}\delta(t)dt=1$

3) Sampling : Multiplication of signal x(t) with impulse with be impulse f^n but strength of impulse is governed by x(t) at location of impulse.

x(t) δ(t –t_0) = x(t_0) δ(t – t_0)

x(t) δ(t) = x(0) δ(t)

4) Shifting :

1) $\int_{-\infty}^{\infty} x(t)\delta(1)\,(t-t_0)dt = x(t_0)$

2) $\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0)$

3) $\int_{-\infty}^{\infty} x(t)\delta(1)(t-t_0)dt = x(t_0)$

if a< t_0< b
0t_0< a
0 t_0> b
Undefined a = b

4) $\delta(t) = \frac{d}{dt}u(t)$

5) Impulse response =
$\frac{d}{dt}$ (step response)
Step response =
$\int_{-\infty}^{t}(\text{impulse response})dt$

6) $u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau = \int_{0}^{\infty}\delta(t-\tau)d\tau$

Simplify the following Questions

1) $t\delta 1.(t) = 0$
2) Sin $t\delta 1.(t) = 0$
3) Cos $t\delta(t-\pi) = -\delta 1.(t-\pi)$
4) $\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$
5) $\delta(3-2t) = \delta - 2(t-3/2) = +\frac{1}{2}\delta(t-3/2)$
6) $e^{-2t}\delta(-3t+2) = e^{-2t}\delta\left[-3\left(t-\frac{3}{2}\right)\right]$

$= \frac{e^{-2+2/3}}{3}\delta(t-2/3)$

7) $(1+2t+3t^2)\delta(\omega+u)$

$= \left\lfloor 1+2(-1)+3(-1)^2\right\rfloor\delta(t+1)$

$= 2\delta 1.(t+1)$

8) $\frac{2-3\omega j}{3+2\omega j}\delta(\omega+4)$

$= \frac{2-3(-4)j}{3+2(-4)j}\delta(\omega+4)$

$= \frac{\sqrt{148}}{\sqrt{73}}\delta(\omega+4)$

9) $\cos(3\tau)\delta\left(\tau-\frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{3}\right)\delta(\tau-\pi/3)$

$= -\delta 1.(\tau-\pi/3)$

10) $\frac{\sin kt}{kt}\delta(t) \to$ L – Hospital rule

$\frac{K\cos kt}{K}\delta(t)\delta = (t)$

11) $\int_{-1}^{1}(3t^2+1)\delta(t)dt = 1$

12) $\int_{-1}^{1}(3t^2+1)\delta 1.(t)\,dt = 1$

13) $\int_{1}^{2}(3t^2+1)\delta(t)\,dt = 0$

14) $\int_{-\infty}^{\infty}(t^2+\cos\,\pi t)\delta(t-1)dt = 0$

15) $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t-1)dt$

$= \frac{1}{2}e^{-t}{}_{t=1} = \frac{1}{2e}$

16) $\int_{-\infty}^{\infty} e^{-t}\delta'(t)dt = -\frac{d}{dt}(e^{-t})\Big|_{t=0} = e^{-t}\Big|_{t=0} = 1$

$\int_{-\infty}^{\infty}\varnothing(t)u'(t)dt = -\int_{-\infty}^{\infty}\varnothing'(t)u(t)dt$

17) $\int_{-\infty}^{\infty}\varnothing(t)\delta'(t)dt = -\varnothing'(0)$

18) $t\delta'(t) = -\delta(t)$

Generalized Derivates : -

$\int_{-\infty}^{\infty}\varnothing(t)(t)dt = (-1^n)\int_{-\infty}^{\infty}\varphi^n(t)\,g(t)\,dt$

g(t) → generalized f^n, $g^n(t)$ → n^{th} derivation
φ(t) → testing f^n, $\varphi^n(t)$ → n^{th} derivation

(1) $\int_{-\infty}^{\infty} e^{-ut}\delta(t+0.3)dt = e^{1.2} = \int_{-4}^{0} e^{-ut}\delta(t+0.3)dt$

(2) $\int_{0}^{10} e^{-ut}\delta(t+0.3)dt = 0$

Example : Step response of LTI System $e^{-3t}u(t)$ find impulse response.

Solution :

$= \frac{d}{dt}[\text{step response}]$

$= \frac{d}{dt}\left[e^{-3t}u(t)\right]$

$= \frac{d}{dt}\left[e^{-3t}u(t) + e^{-3t}u'(t)\right]$

$= -3e^{-3t}u(t) + e^{-3t}\delta(t)$

$= -3e^{-3t}u(t) + \delta(t)$

Discrete time impulse: Unlike δ (t), δ (n) does not represent area under the signal rather it represent the amplitude.

δ (n) = 0 for n ≠ 0

1 for n = 0

Properties of δ (n)

1. δ (an) = δ(n) **
2. x(n) δ (n – k) = x(k) δ (n – k)
3. δ (n) = u(n) – u(n – 1)
4. u(n) = $\sum^{n}\delta(k)$

   k = + ∞

   $\sum_{k=0}^{k=+\infty} {}^{\infty}\delta(n-k) = \sum_{k=-\infty} {}^{n}\delta(k)$

   there fore step response = $\sum_{k=-\infty} {}^{\infty}$impulse response

Example : Step response of LTI system is given by $\left(-\frac{2}{3}\right)^n u(n)$, find the impulse response.

Impulse response

$= S(n) - S(n-1)$

$= \left(-\frac{2}{3}\right)^n u(n) - \left(-\frac{2}{3}\right)^{n-1} u(n-1)$

$= \left(-\frac{2}{3}\right)^n u(n) + \left(-\frac{2}{3}\right)^n \times \frac{3}{2}u(n-1)$

$= \left(-\frac{2}{3}\right)^n \left[u(n) + \frac{3n}{2}u(n-1)\right]$

1.4.4 EXPONENTIAL FUNCTION

The complex exponential signal is defined as x(t) = e^{st} where s = σ + jω, a complex number. Then this signal x(t) is known as a general complex exponential signal whose real part $e^{\sigma t}$ cos ωt and imaginary part $e^{\sigma t}$ sin ωt are exponentially increasing (σ > 0) or decreasing σ , 0 respectively. If $_s$ = σ, then x(t) = eσt which is a real exponential signal. If (σ > 0), then x(t) is a growing exponential; and if (σ < 0), then x(t) is a decaying exponential.

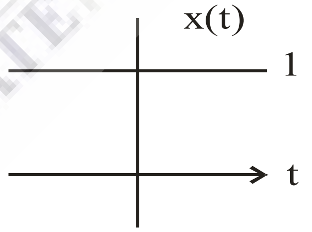
Continuous Time

General form x(t) = e^{st}

S = σ + jω

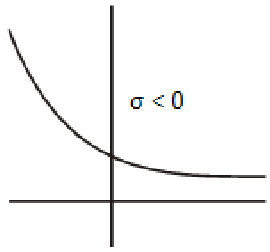
Case 1. σ = 0, ω = 0, s = 0

x(t) = 1 dc

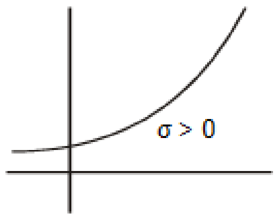


Case 2. σ ≠ 0, ω = 0, s = σ

x = $e^{\sigma t}$



Decay exponential

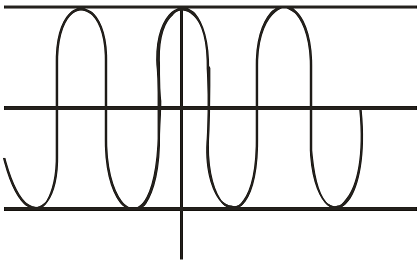


Growing exponential

Case 3. $\sigma = 0, \omega \neq 0, \quad s = j\omega t$

$x(t) = e^{+j\omega t} = \cos \omega t + j \sin \omega t$

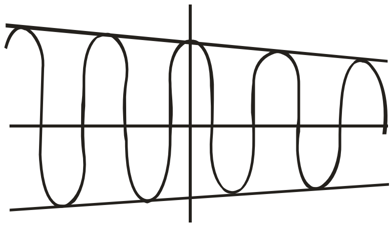
Will oscillate from +1 to -1



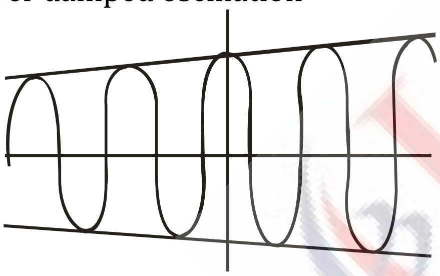
Case 4. $\sigma \neq 0, \omega \neq 0, \quad s = \sigma + j\omega$

$x(t) = e^{0t}e^{j\omega t}$

(i) $\sigma < 0$ $x(t) = e^{0t}e^{j\omega t}$ Under damped oscillation



(ii) Over damped oscillation



$\sigma > 0$. $x(t) = e^{0t}e^{j\omega t}$

1.5 EVEN AND ODD SIGNAL

If a signal exhibits symmetry in the time domain about the origin, it is called an even signal. The signal must be identical to its reflection about the origin. Mathematically, and even signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

An odd signal exhibits anti-symmetry. The signal is not identical to its reflection about the origin, but to its negative. An odd signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

$x_1(t) = \sin \omega t$ and $x_2(t) = \cos \omega t$ are good examples of odd and even signal, respectively. An even signal which often occurs in the analysis of signals is the sinc function.

A signal can be expressed as a sum of two components, namely, the even component of the signal and the odd component of the signal. The even and odd components can be obtained from the signal itself.

$x(t) = x_{even}(t) + x_{odd}(t)$

$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)]$ and

$x_{odd}(t) = \frac{1}{2}[x(t) + x(-t)]$

A signal x(t) or x[n] is an even signal if

$x(-t) = x(t)$

$x[-n] = x[n]$

an odd signal if $x(-t) = -x(t)$

$x[-n] = -x[n]$

Any signal x(t) or x[n] can be expressed as

$x(t) = x_e(t) + x_o(t)$

$x[n] = x_e[n] + x_o[n]$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ even part

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$ even part

$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$ odd part

$x_0(n) = \frac{1}{2}[x(n) - x(-n)]$ odd part

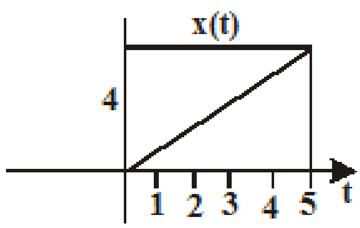
Note:- The product of

Two even or odd signal = an even signal

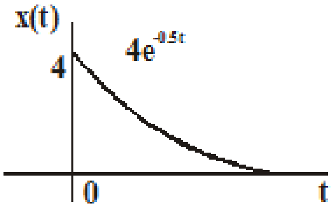
An even signal an odd signal = an odd signal

Example :Sketch even and odd components of signals

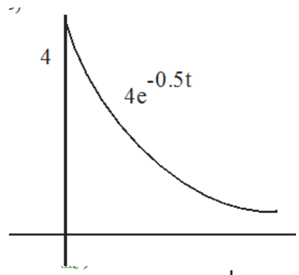
(a)

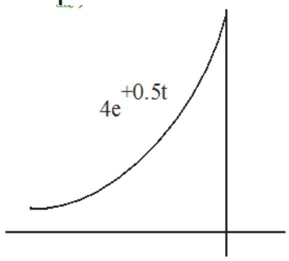


(b)

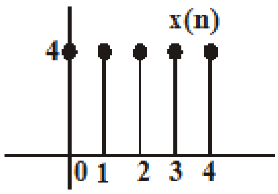


(b)

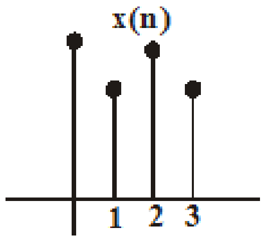




(c)

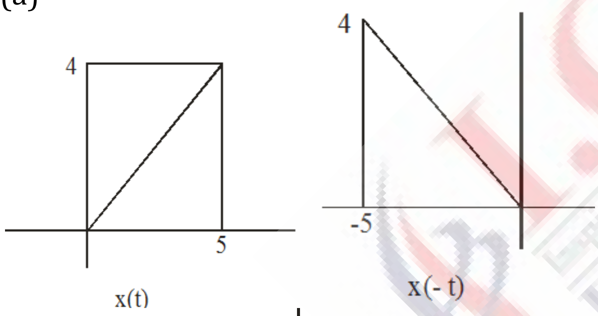


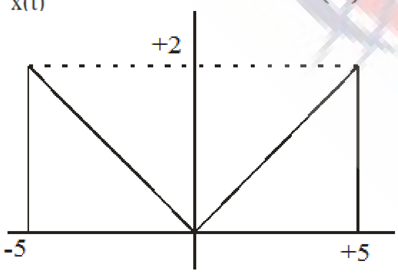
(d)



Solution :

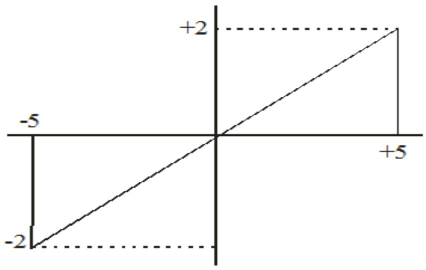
(a)



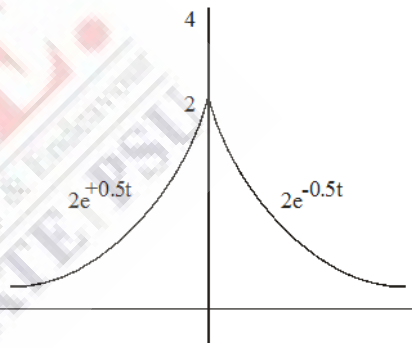


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_0(t) = \frac{x(t) - x(-t)}{2} =$$

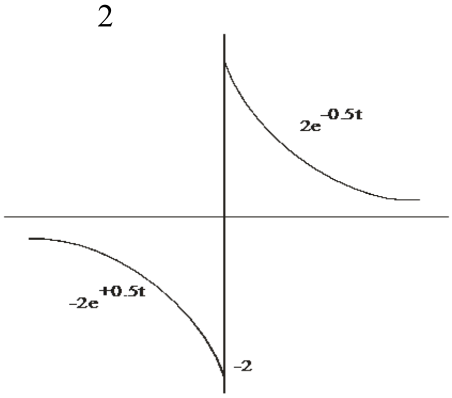


x(t)
x(-t)

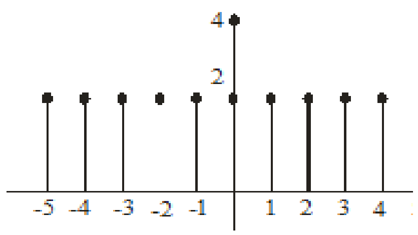


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_0(t) = \frac{x(t) - x(-t)}{2} =$$



(c)



GATE QUESTIONS (EC)

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GATE QUESTIONS (IN)

Topics **Page No**

GATE QUESTIONS
(ELECTRONICS)

1 BASICS OF SIGNAL AND SYSTEM

Q.1 Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

a) 1 b) -1

c) 0 d) $\frac{\pi}{2}$

[GATE-2001]

Q.2 If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

a) E b) $\frac{E}{2}$

c) 2E d) 4E

[GATE-2001]

Q.3 Let P be linearity, Q be time – invariance, R be causality and S be stability .A discrete- time system has the input- output relationship.

$$y(n)\begin{cases} x(n) & n \geq 1 \\ 0, & n = 0 \\ x(n+1), & n \leq -1 \end{cases}$$

Where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties.

a)P,S but not Q,R b)P,Q,S but not R

c)P,Q,R,S d)Q,R,S but not P

[GATE-2003]

Q.4 Consider the sequence $x[n] = [-4, j5 \;\; \underset{\uparrow}{1+j2} \;\; 4]$

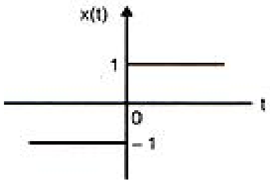
The conjunction antisymmetric part of the sequence is

a)$[-4 - j2.5 \; j2 \; 4 - j2.5]$ b)$[-j2.5 \; 1 \; j2.5]$

c)$[-j5 \quad j2 \quad 0]$ d)$[-4 \quad 1 \quad 4]$

[GATE-2004]

Q.5 The function $x(t)$ is shown in the figure .Even and odd parts of an odd parts of a unit –step function $u(t)$ are respectively.



a)$\frac{1}{2}, \frac{1}{2}x(t)$ b)$-\frac{1}{2}, \frac{1}{2}x(t)$

c)$\frac{1}{2}, -\frac{1}{2}x(t)$ d)$-\frac{1}{2}, -\frac{1}{2}x(t)$

[GATE-2005]

Q.6 The power in the signal $s(t) = 8\cos(20\pi t - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

a) 40 b) 41

c) 42 d) 82

[GATE-2005]

Q.7 The Dirac delta function δ(t) is defined as

a) $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

b) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

c) $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

d) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

[GATE-2006]

Q.8 A system with input $x[n]$ and output $y[n]$ is given as $y[n] = \left(\sin\frac{5}{6}\pi n\right) \times (n)$

The system is

a) linear, stable and invertible

b) non-linear stable and non-invertible

c) linear stable and non- invertible

EXPLANATIONS

Q.1 **(a)**

$\int_{-\infty}^{\infty} \delta(t) \left(\frac{3t}{2}\right) dt = f(0)$

$= \cos\left(\frac{3\times0}{2}\right) = \cos 0 = 1$

Q.2 **(b)**

$E = \int_{-\infty}^{\omega} f(t)^2 dt$

$E' = \int_{-\infty}^{\omega} f(2t)^2 dt$

$= \int_{-\infty}^{\infty} f(p)^2 \frac{dp}{2} (2t = p; dt = \frac{dp}{2})$

$E' = \frac{E}{2}$

Q.3 **(a)**

$y(n - n_0)x(n - n_0 + 1)$

(time varying)

$y(n) = x(n + 1)$

(depends on future)

ie., $y(1) = x(2)$ (non causal)

For bounded input, system has bounded output, So it is stable.

$y(n) = x(n), n \geq 1$

$= 0, n = 0$

$= x(n+1), n \leq -1$

So, system is linear.

$= Lt_{s\to\infty} \cdot \frac{5\times s}{s(s^2+3s+2)} = 0$

Q.4 **(a)**

$x(n) = [-4 - j5, 1 + 2j, 4]$

$x^*(-n) = [4 \;\; 1 - 2j, -4 + j5]$

$x_{CAS}(n) = \frac{x(n) - x^*(-n)}{2}$

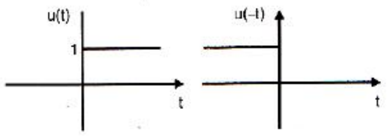
$= [-4 - 2.5j, 2j, 4 - j2.5]$

Q.5 **(a)**

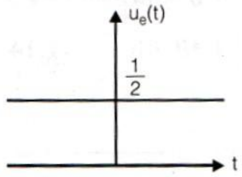
Even part = $\frac{\alpha(t)+\alpha(-t)}{2}$

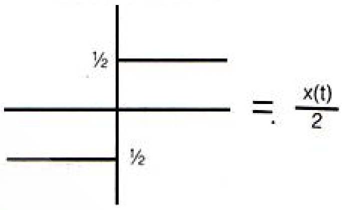
Odd part = $\frac{\alpha(t)+\alpha(-t)}{2}$

Here $\alpha(t) = u(t)$



$u_e(t) = \frac{1}{2}$





Q.6 **(a)**

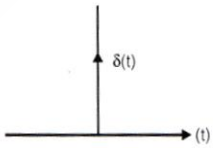
$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right)$

$4 \sin 15\pi t$

$= 8 \sin 20\pi t + 4 \sin 15\pi t$

$P = \frac{8^2}{2} + \frac{4^2}{2} = 32 + 8 = 40$

Q.7 **(d)**



Q.8 **(c)**

$y[n] = \left(\sin \frac{5}{6}\pi n\right) \times (n)$

Let $x[n] = \delta(n)$

$\therefore y[n] = \sin 0 = 0 (bounded)$

BIBO stable

Q.9 **(c)**

A system is causal if the output at any time depends only on values of the input at the present time and in the past.

Q.10 **(a)**

Q.11 **(d)**

$y = \int_{-\infty}^{t} x(\tau) \cos(3\tau)\, d\tau$

$y(t - t_0) = \int_{-\infty}^{t-t_0} x(\tau) \cos(3\tau)\, d\tau$

GATE QUESTIONS
(ELECTRICAL)

ASSIGNMENT QUESTIONS

Q.1 An input x[n] with length 3 is applied to a linear time invariant system having an impulse response h[n] of length 5, and Y(ω) is the DTFT of the output y[n] of the system. If |h[n]| ≤ L and |x[n]| ≤ B for all n, the maximum value of Y(0) can be

a) 15 LB b) 12 LB
c) 8 LB d) 7 LB

Q.2 The two-sided Laplace transform of $x(t)=e^{-3t}u(t)+e^{2t}u(-t)$ is

a) $X(s)=\frac{-5}{s^2+s-6}, -3<\sigma<2$

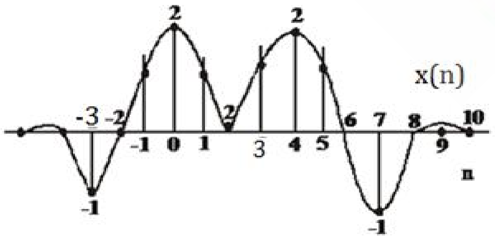
b) $X(s)=\frac{-5}{s^2+s-6},\ 2<\sigma<3$

c) $X(s)=\frac{-5}{s^2+s-6},\ -3<\sigma<-2$

d) $X(s)=\frac{-5}{s^2+s-6},\ -3<\sigma<-2$

Q.3 For the signal x[n] shown in figure $x[n]=0$ for n < -3 and n > 7

If X(ω) is the Fourier transform of x[n], which one of the following is TRUE?



a) X (0) = 5

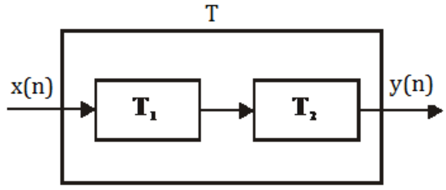
b) $\int_{-\pi}^{\pi} x(\omega)d\omega = 2\pi$

c) the phase $\angle X(\omega)$ odd function

d) X(ω) = X(-ω)

Q.4 Two systems T_1 and T_2 are cascaded to get the system T as shown in fig

Which one of the following statements is TRUE?



a) If both T_1 and T_2 are linear then T is NOT necessarily linear

b) If both T_1 and T_2 are time invariant then T is NOT necessarily time invariant

c) If both T_1 and T_2 are non-linear then T is NOT necessarily non-linear

d) If both T_1 and T_2 are causal then T is NOT necessarily causal

Q.5 If $x[n]=\begin{cases}\frac{2}{\pi}, & n=0\\ \frac{\sin 2n}{\pi n}, & n\neq 0\end{cases}$, the energy of x[n] is

a) $\frac{2}{\pi}$ b) $\frac{1}{\pi}$

c) $\frac{1}{2\pi}$ d) $\frac{3}{\pi}$

Q.6 The pole-zero plot of the transfer function ($H_a(s)$) of a linear time invariant system in s-plane is shown in Fig. The corresponding impulse response $h_a(t)$ is sampled at 2Hz to get the discrete-time impulse response sequence h[n]. If the right half of the s-plane is mapped into the outside of the unit circle, which one of the following shows the equivalent pole-zero plot of H(z) in the z-plane (the concentric circles are $|z|=\frac{1}{e}$ and $|z|=1$)?

EXPLANATIONS

Q.1 (d)

x(n) → M = 3

h(n) → N = 5

y(n) → M + N – 1 = 7

DTFT of the output y[n] → Y(ω)

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y[e^{j\Omega}] = \sum_{k=-\infty}^{\infty} |h(k)|\, |x(k)|\, e^{-j\Omega n}$$

$$Y[0]_{max.} = \sum_{k=-\infty}^{\infty} L.B$$

Y(0) → 7 (|h[n]| ≤ L X |x[n]| ≤ B) = 7LB

Q.2 (a)

$x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

two-sided Laplace transform

$e^{-3t}u(t)$ + $e^{2t}u(-t)$

↓ ↓

$\frac{1}{s+3}$, Re(s) > - 3 $-\frac{1}{s-2}$, Re(s) < 2

- 3 < Re (s) < 2

$$\frac{1}{s+3} - \frac{1}{s-2}$$

$$\frac{-5}{s^2+s-6}$$

Q.3 (c)

$$\text{(a) } X(0) = \sum_{n=-3}^{7} x(n)$$

= -1 + 0 + 1 + 2 + 1 + 0 + 1 + 2 + 1

X(0) = 6 False

$$\text{(b) } x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\, e^{j\Omega n}\, d\Omega$$

$$2\pi\, x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\, d\Omega$$

= 2π x 2 = 4π False

(c) X(ω) = X(-ω)

|X(ω)| → even function

$\angle X(\omega)$ → odd function

True

(d) False

Q.4 (a)

Multiplication of two linear system T_1 and T_2 result will be non linear

Q.5 (a)

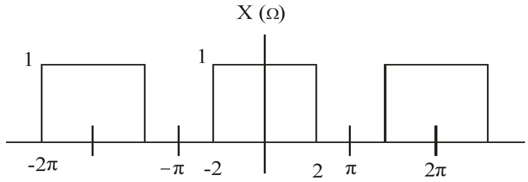
If
$$x[n] = \begin{cases} \frac{2}{\pi}, & n = 0 \\ \frac{\sin 2n}{\pi n}, & n \neq 0 \end{cases}$$

$$x[n] = \begin{cases} \frac{2}{\pi}; & n = 0 \\ \frac{2}{\pi}\frac{\sin 2n}{\pi n}; & n \neq 0 \end{cases}$$

$$x(n) \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega})$$

$$\frac{\sin Wn}{\pi n} \overset{DTFT}{\longleftrightarrow} X(\Omega) = \begin{cases} 1 & |\Omega| \leq w \\ 0 & W \leq |\Omega| \leq \pi \end{cases}$$

$$\frac{\sin 2n}{\pi n} \leftrightarrow X(\Omega) = \begin{cases} 1 & |\Omega| \leq 2 \\ 0 & 2 \leq |\Omega| \leq \pi \end{cases}$$



By parsevel's Theorems

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \int_{-W}^{W} 1(d\Omega)$$

$$= \frac{1}{2\pi}\int_{-2}^{2} d\Omega$$

$$E = \frac{1}{2\pi} \times 4 = \frac{2}{\pi}$$

Q.6 (b)

$$H_a(S) = \frac{k(s+2)\left(s^2 + \frac{\pi^2}{4}\right)}{s(s+2)+\pi^2}$$