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POWER SYSTEMS

For

ELECTRICAL ENGINEERING

POWER SYSTEMS

SYLLABUS

Basic power generation concepts; transmission line models and performance; cable performance, insulation; corona and radio interference; distribution systems; per-unit quantities; bus impedance and admittance matrices; load flow; voltage control; power factor correction; economic operation; symmetrical components; fault analysis; principles of over-current, differential and distance protection; solid state relays and digital protection; circuit breakers; system stability concepts, swing curves and equal area criterion; HVDC transmission and FACTS concepts.

ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	4	11	26
2004	6	9	24
2005	4	7	18
2006	4	10	24
2007	4	9	22
2008	3	8	19
2009	2	5	12
2010	3	5	13
2011	5	4	13
2012	3	2	7
2013	1	3	7
2014 Set-1	3	5	13
2014 Set-2	2	4	10
2014 Set-3	3	3	9
2015 Set-1	1	4	9
2015 Set-2	2	3	8
2016 Set-1	2	4	10
2016 Set-2	4	3	10
2017 Set-1	2	3	8
2017 Set-2	3	3	9

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1 TRANSMISSION LINE MODEL & PERFORMANCE

1.1 INTRODUCTION

An electric transmission line can be represented by a series combination of resistance, inductance and shunt combination of conductance and capacitance. These parameters are symbolized as R, L, G and C respectively. Of these R and G are least important in the sense that they do not affect much the total equivalent impedance of the line and hence the transmission capacity.

The effective resistance is equal to the d.c. resistance of the conductor only if the current is uniformly distributed throughout the section of the conductor. The loss on the overhead line is due to (i) ohmic loss in the power conductors, (ii) corona los and (iii) leakage at the insulators

1.1.1 MAGNETIC FLUX DENSITY

A current carrying conductor produces a magnetic field which is in the form of closed circular loops around the conductor.

$$H = \frac{B}{\mu}$$

1.1.2 INDUCTORS AND INDUCTANCE

An inductor is a device which stores energy in a magnetic field. By definition, the inductance L of an inductor is the ratio of its total magnetic flux linkages to the current I through the inductor or

$$L = \frac{N\Psi_m}{I} = \frac{\lambda}{I}$$

For a medium permeability is constant ferrous media

$$L = \frac{d\lambda}{dI}$$

1.1.3 MAGNETIC FIELD INTENSITY DUE TO A LONG CURRENT CARRYING CONDUCTOR

The current is uniformly distributed across the section of the conductor. The flux linkages here will be both due to internal flux and external flux.

Cylinder with radius r < R

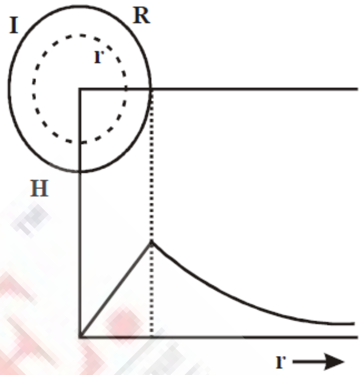


Fig. Variation of H due to current in the conductor for r ≤ R and r > R

$$I' = I\left(\frac{r}{R}\right)^2$$

$$H_r = \frac{Ir}{2\pi R^2}$$

A cylinder with radius r > R

$$H_r = \frac{I}{2\pi r}$$

1.2 FLUX LINKAGES

1.2.1 INTERNAL FLUX LINKAGES

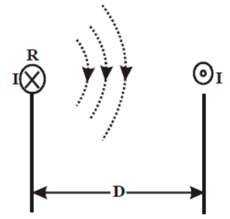


Fig. Magnetic field due to one conductor of a 1-ϕ transmission line

$$H = \frac{Ir}{2\pi R^2}$$

$$B = \mu H = \mu_0 H$$

$$= \frac{\mu_0 I}{2\pi R^2}.r\,(\text{as } \mu_r = 1)\text{ for conductors}$$

$d\varphi = B$. Area normal to flux density B

= B.dr.l

Total internal flux linkages $\lambda = \int_0^R d\lambda$

$$= \frac{\mu_0 I}{2\pi R^4}\int_0^R r^3 dr = \frac{\mu_0 I}{8\pi}$$

1.2.2 EXTERNAL FLUX LINKAGES

$R \le r < D$

The total external flux linkages due to current flow in one conductor

$$\lambda = \int_R^{D-R} d\lambda$$

$$\lambda = \frac{\mu_0 I}{2\pi}\ln\frac{D}{R}$$

Total flux linkages due to one conductor

= Total internal flux linkage + Total external flux linkages

$$= \frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi}\ln\frac{D}{R}$$

Total flux linkage due to both the conductors $= 2\left[\frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi}\ln\frac{D}{R}\right]$

Inductance L per unit length

$$= \left[\frac{\mu_0}{4\pi} + \frac{\mu_0}{2\pi}\ln\frac{D}{R}\right]\text{Henry / metre}$$

Since $\mu_0 = 4\pi\times10^{-7}$,

$$L = \left[1 + 4\ln\frac{D}{R}\right]\times10^{-7}\,\text{Henry / metre}$$

$$= 4\times10^{-7}\times\left[\frac{1}{4} + \ln\frac{D}{R}\right]\text{Henry / metre}$$

Since $\ln e^{1/4} = \frac{1}{4}$

$$\therefore L = 4\times10^{-7}\left(\ln e^{1/4} + \ln\frac{D}{R}\right)$$

$$= 4\times10^{-7}\ln\frac{D}{Re^{-1/4}}$$

$$= 4\times10^{-7}\ln\frac{D}{R'}\,\text{Henry / metre}$$

1.2.3 FLUX LINKAGES OF ONE CONDUCTOR IN A GROUP OF CONDUCTORS

Find out the flux linkages of one conductor due to current flowing in the conductor self and the current flowing in the other conductors. It is assumed here that the sum of the currents in various conductors is zero.

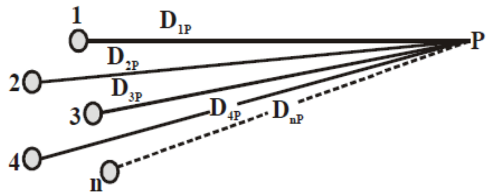


Fig. Cross-sectional view of a group of n conductors Point P is remote from the group of conductors

Assume here that P is a point very far from the group of the conductors. The objective here is to calculate the flux linkages of say conductor 1 due to the current I_1, carried by the conductor itself and flux linkage to conductor 1 due to the current carried by conductors 2, 3,…….n.

Due to the current I_1

$$\lambda_{1p_1} = \frac{\mu_0 I_1}{8\pi} + \frac{\mu_0 I_1}{2\pi}\ln\frac{D_{1p}}{R_1}$$

Due to current in conductor 2 are

$$\lambda_{1p_2} = 2\times10^{-7}.I_2\ln\frac{D_{2p}}{D_{12}}$$

Since $I_1 + I_2 + \ldots\ldots + I_n = 0$,

The net flux linkages λ_{1p}

$$\lambda_{1p} = 2\times10^{-7}\left[I_1\ln\frac{1}{R_1} + I_2\ln\frac{1}{D_{12}} + \ldots\ldots + I_n\ln\frac{1}{D_{1n}}\right]$$

wb-turns/metre

1.3 INDUCTANCE OF TWO-WIRE (1-ø) TRANSMISSION LINE

1.3.1 INDUCTANCE OF 3-ϕ UNSYMMETRICALLY SPACED TRANSMISSION LINE

$a \ne b \ne c$ and each has a radius of R metres

$$\lambda_a = 2\times10^{-7}\left[I_a\ln\frac{1}{R`} + I_b\ln\frac{1}{c} + I_c\ln\frac{1}{b}\right]$$

$$\lambda_b = 2\times10^{-7}\left[I_a\ln\frac{1}{c} + I_b\ln\frac{1}{R'} + I_c\ln\frac{1}{a}\right]$$

$$\lambda_c = 2\times10^{-7}\left[I_a\ln\frac{1}{b} + I_b\ln\frac{1}{a} + I_c\ln\frac{1}{R'}\right]$$

I_a as reference

$$I_b = k^2 I_a \text{ and } I_c = kI_a$$
$$\text{where } k = (-0.5 + j0.866)$$

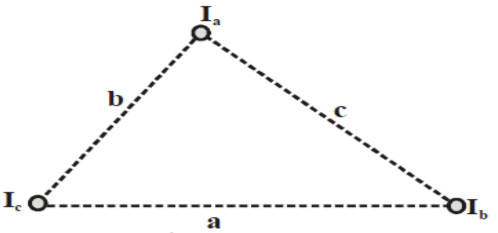


Fig. 3-ϕ transmission line with unsymmetrical spacing

In case the transmission line is transposed i.e., each conductor takes all the three position of the conductors

$$L = \frac{L_a + L_b + L_c}{3}$$

$$= \frac{1}{3}\left[2\times10^{-7}\left(3\ln\frac{1}{R'} - \ln\frac{1}{abc} - j\frac{\sqrt{3}}{2}\ln 1\right)\right]$$

$$= 2\times10^{-7}\ln\frac{3\sqrt{abc}}{R'}\text{Henry / metre}$$

$$L = 2\times10^{-7}\ln\frac{d}{R'}\text{Henry / metre}$$

1.3.2 INDUCTANCE OF COMPOSITE CONDUCTORS

The current is assumed to be equally divided amongst the strands. One group of conductors act as a 'go' conductor for the single-phase line and the other as the 'return'. The current per strand is I/m ampere in one group and –I/n ampere in the other.

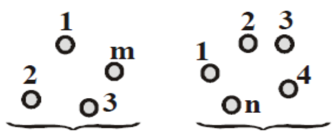


Fig. Inductance of composite conductor-1-ϕtransmission line

$$L_{av} = \frac{L_1 + L_2 + \ldots\ldots + L_m}{m}$$

Since all the strands of conductor A are electrically parallel, the inductance of conductor will be

$$L_A = \frac{L_{av}}{m} = \frac{L_1 + L_2 + \ldots\ldots\ldots + L_m}{m^2}$$

Substituting the values of L_1, L_2,………..,L_m in equation

$$L_A = 2\times10^{-7}\ln\frac{\sqrt[mn]{(D'_{11}D'_{12}\ldots D_{1n})(D'_{21}D'_{22}\ldots D'_{2n})\ldots(D'_{m1}D'_{m2}\ldots D'_{mn})}}{\sqrt[m^2]{(R'D_{12}D_{13}\ldots D_{1m})(R'D_{21}D_{23}\ldots D_{2m})\ldots(R'D_{m1}D_{m2}\ldots D_{mn})}}$$

The mnth root of the product of the mn distances between m strands of conductor A and n strands of conductor B is called geometric mean distance (GMD) and is denoted as D_m and the m^2th root of m^2 distance i.e., the distance of the various strands from one of the strands and the radius of the same strand, geometric mean radius (GMR) or self GMD.

$$L = L_A + L_B$$

1.3.3 INDUCTANCE OF DOUBLE 3-Φ LINE

Conductors α and α' are electrically parallel and constitute one phase.
The conductors of two phases are placed diagonally opposite rather than in the same horizontal plane, in all the three positions. By doing this the self GMD of the conductors is increased whereas the GMD reduced, thereby the inductance per phase in lowered.

1.4 TRANSPOSITION OF POWER LINES

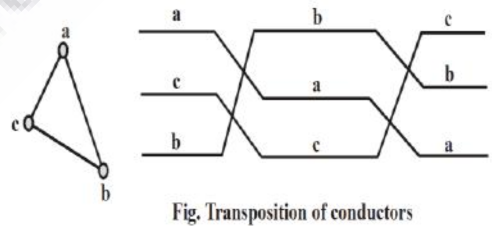


Fig. Transposition of conductors

By transposition of conductors is meant the exchanging of position of the power conductors at regular intervals along the line, so that each conductor occupies the original position of every other conductor over an equal distance.
If the spacing is unsymmetrical even though the system operates under balanced condition, voltage drops of different magnitude will be there in the three conductors due to unequal inductance of the three phases. The magnetic field external to the conductors is not zero, thereby causing induced voltages in adjacent electrical circuits, particularly telephone lines that may result in

telephone interferenceIt is enough to transpose either power line or the communication lines. Under balanced operating condition, the magnetic field linking an adjacent telephone line is shifted 120° in time phase with each rotation of the conductor positions in the net voltage induced in the telephone line is zero as it is the sum of three induced voltage which are displaced by 120° in time phase.
The transposition, however, may be effected at the intermediate switching station.

1.5 COMPOSITE CONDUCTORS

For transmission lines operating at high voltages normally stranded conductors are used. These conductors are known as composite conductors as they compose of two or more elements or strands electrically in parallel. By using different Steel cored, reinforced aluminium conductors (ASCR) which combine the lightness, electriJcal conductivity and restlessness of aluminum with the high tensile strength of steel

1. Aluminium conductors steel reinforced cheaper
2. The superior mechanical strength
3. A reduction in the number of supports
4. The increase in span length5. Corona losses reduced

1.6 CONCEPT OF GEOMETRIC MEAN DISTANCE

Geometric mean distance is a mathematical concept used for the calculation of inductance.
On the circle is

$$GMD_p = 5\sqrt{D_1D_2D_3D_4D_5}$$

Number of points on the circle are increased to infinity, the distance between the point P and centre of the circle.
The GMD between two circular areas will be the distance between the centres of the two areas and so on. For voltages in excess of 230 kV. It is preferable to use more than one conductor per phase which is known as bundling of conductors. A bundle conductor is a conductor made up of two or more sub-conductors and is used as one phase conductor.
The advantages in using bundle conductors

i) Reduced reactance
ii) Reduced voltage gradient
iii) Reduced corona loss
iv) Reduced radio interference
v) Reduced surge impedance

The self GMD of the conductors is increased.

$$\text{Reactance} = K \ln \frac{GMD}{GMR}$$

Since the voltage gradient is reduced by using bundled conductors the radio interference is also reduced.
Surge impedance $\sqrt{L/C}$. Since by bundling, the self GMD is increased, the inductance is reduced and capacitance increased, as a result the surge impedance is reduced. The maximum power that can be transmitted is increased.
The basic difference between a composite conductor and bundled conductor is that the sub-conductors of a bundled conductor are separated from each other by a distance of almost 30 cms or more and the wires of a composite conductor touch each other.

1.7 SKIN AND PROXIMITY EFFECT

When direct current flows in the conductor, the current is uniformly distributed across the section of the conductor whereas flow of alternating current is non-uniform, with the outer filaments of the conductor carrying more current than the filaments closer to the centre.
A higher resistance to alternating current than to direct current and is commonly known as skin effect. This effect is more, the more is the frequency of supply and the size of the conductor.
The flux linkages per ampere to inner strands is greater than those of outer strands. Hence the inductance/impedance of the inner strands is greater than those of

outer strands which results in more current in the outer strands as compared to the inner strands. This non-uniformity of flux linkage is the main cause of skin effect.

The alternating magnetic flux in a conductor caused by the current flowing in a neighboring conductor gives rise to circulating currents which cause an apparent increased in the resistance of a conductor. This phenomenon is called proximity effect. In a two-wire system more lines of flux link elements farther apart than the elements nearest each other. Therefore, the inductance of the elements farther apart is more as compared to the elements near each other and the current density is less in the elements farther apart than the current density in the elements near each other. The effective resistance is, therefore, increased due to non-uniform distribution of current. The proximity effect is pronounced in case of cables where the distance between the conductors is small whereas for overhead lines with usual spacing the proximity effect is negligibly small.

1.8 CAPACITANCE OF TRANSMISSION LINES

The flow of current through a conductor gives rise to a magnetic field and charging of conductor results in an electric field. A charge if brought in the vicinity of this electric field experiences a force as electric field intensity E. Newton per coulomb or volts per metre.

$$E_r = \frac{\rho L}{2\pi\varepsilon_0 r}$$

$$V = \frac{\rho L}{2\pi\varepsilon_0} \ln \frac{r_2}{r_1}$$

1.8.1 CAPACITANCE OF 1-Φ TRANSMISSION LINE

The charge ρ_L coulomb/metre is distributed on the surface of the conductor which is non-uniformly distributed over the surface such that it has higher density on the adjacent sides of the conductors. If operating voltage V, distance of separation h and radius of the equipotential surface r.

$$E_r = \frac{\rho L}{\pi\varepsilon_0} \ln \frac{h}{r}$$

$$C = \frac{\pi\varepsilon_0}{\ln h / r} \text{ F / metre}$$

Equation for inductance contains a constant term corresponding to the internal flux linkages whereas since charges reside on the surface of the conductor, similar term is absent in the capacitance expression.

The concept of self GMD is applicable for inductance calculation and not for the capacitance. The capacitance between one conductor and a neutral point

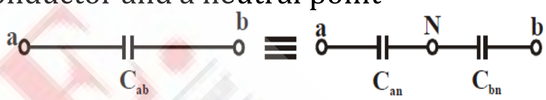


Fig.

$$C_{an} = 2C_{ab} = \frac{2\pi\varepsilon_0}{\ln \frac{h}{r}}$$

1.8.2 CAPACITANCE OF A 3-PHASE UNSYMMETRICALLY SPACED TRANSMISSION LINE

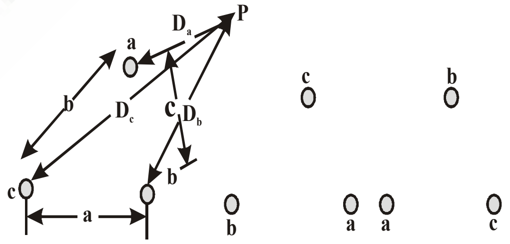


Fig. Unsymmetrically spaced transposed 3-phase transmission line

For an untransposed line the capacitances between conductors to neutral of the three conductors are unequal. In transposed lines the average capacitance of each conductor to neutral is the same as the capacitance to neutral of any other phase.

3-phase balanced system, ρ_α reference charge,

$$V_a' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a}{r} + \rho_b \ln \frac{D_b}{c} + \rho_c \ln \frac{D_c}{b}\right]$$

$$V_a^{''} = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln\frac{D_b}{r} + \rho_b \ln\frac{D_c}{a} + \rho_c \ln\frac{D_a}{c}\right]$$

$$V_a^{'''} = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln\frac{D_c}{r} + \rho_b \ln\frac{D_a}{b} + \rho_c \ln\frac{D_b}{a}\right]$$

Average

$$V_a = \frac{V_a^{'} + V_a^{''} + V_a^{'''}}{3}$$

$$\rho_a + \rho_b + \rho_c = 0$$

$$V_a = \frac{\rho_\alpha}{2\pi\varepsilon_0}\ln\frac{\sqrt[3]{abc}}{r} = \frac{\rho_\alpha}{2\pi\varepsilon_0}\ln\frac{GMD}{r}$$

$$C = \frac{\rho_\alpha}{V_a} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD}{r}} F/\text{metre}$$

For a symmetrical spacing of the conductors, a = b = c = h

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{h}{r}}$$

1.8.3 EFFECT OF EARTH ON THE CAPACITANCE OF CONDUCTORS

The electric flux lines and the equipotential lines are orthogonal to each other. The earth is considered to be conducting and an equipotential plane of infinite extent. The positive charge on the conductor induces negative charges on the earth surface. This distribution of charge on the surface of the earth should be replaced by an equivalent charge for the calculation of electric field potential and other related quantities due to this isolated charged conductor.

Since earth is an equipotential plane which is possible only if we assume the presence of an imaginary conductor below the surface of the earth at a depth equal to the height of the actual conductor above the surface of the earth.

1.8.4 CAPACITANCE OF SINGLE CONDUCTOR

The single conductor with the earth is equivalent to a single-phase transmission line.

$$C = \frac{\pi\varepsilon_0}{\ln\frac{D_{ab}}{r}\frac{D_{aa'}}{D_{ab'}}}$$

Ratio $\frac{D_{aa'}}{D_{ab'}} < 1$, the effect of earth on capacitance of the system is to increase it.

The effect of earth is to increase the capacitance.

1.9 PERFORMANCE OF LINES

By performance of lines determination of efficiency and regulation of lines

The efficiency of lines is defined as

$$\%\text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power sent from the sending end}} \times 100$$

$$\%\text{efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end} + \text{losses}} \times 100$$

Receiving end and sending end

The regulation change in the receiving end voltage, expressing in per cent of full load voltage, from no load to full load, the sending end voltage and frequency constant

$$\%\text{regulation} = \frac{V_r^{'} - V_r}{V_r} \times 100$$

$V_r^{'}$ is the receiving end voltage under no load condition and $V_r^{'}$ the receiving end voltage under full load condition.

1.9.1 REPRESENTATION OF LINES

Four distributed parameters, series resistance and inductance, and shunt capacitance and conductance

Electrical power transmitted at approximately the speed of light. One full wave variation of voltage or current

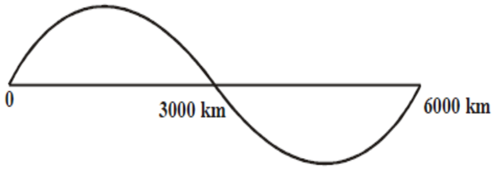


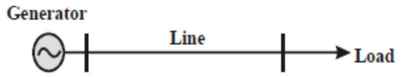
Fig. Voltage distribution of 50 Hz supply

$$f.\lambda = \upsilon$$

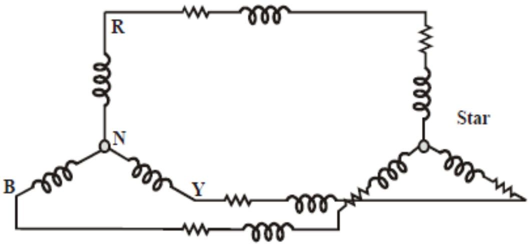
$$f = 50 \text{ and } \upsilon = 3\times10^8 \text{ m/sec.}$$

$$\lambda = \frac{v}{f} = \frac{3\times10^8}{50} = 6\times10^6 \text{metres}$$

=6000 Km



(a) Single-line diagram of a 3-phase system



(b) 3-phase diagram of (a)

Less than about 160 km, the voltage or current variation on the line is not much parameters assumed to be lumbed and not distributed.

Short categorized as short transmission lines and medium transmission lines. The lines upto about 80 km short effect of shunt capacitance is neglected above 80 and below 160 km as medium length the shunt capacitance can be assumed to be lumped at the middle of the line or half of the shunt capacitance may be considered to be lumped at each end of the line. Nominal-T and nominal-π respectively, more than 160 km the parameters are distributed and rigorous calculations are required.

The 3-phase system is normally balanced system irrespective of the fact that the conductors are not transposed, as the untransposed conductors introduce slight dissymmetry which can be ignored for all practical purposes.

The currents in a balanced polyphase network is zero and, therefore, the current through the wire connected between the star point of the load and neutral of the system is zero. The star point of the load and neutral of the system are at the same potential.

A 3-phase balanced system can, therefore, be analysed on single-phase basis in which the neutral wire is of zero impedance.

A 3-phase balanced system can, therefore, be analysed on single-phase basis in which the neutral wire is of zero impedance.

1.9.2 SHORT TRANSMISSION LINE

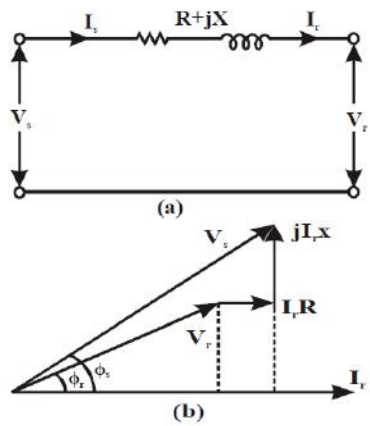


Fig. Short-transmission line (a) equivalent circuit,

$$V_s \cos\varphi_s = V_r \cos\varphi_r + I_r R$$

$$V_s \sin\varphi_s = V_r \sin\varphi_r + I_r X$$

Squaring and adding

$$V_s^2 = V_r^2 + 2I_r R V_r \cos\varphi_r + 2I_r X V_r \sin\varphi_r + I_r^2(R^2+X^2)$$

Last term

$$V_s \approx V_r + I_r R\cos\varphi_r + I_r X \sin\varphi_r$$

$$V_r' = V_s$$

$$\%\text{regulation} = \frac{V_s - V_r}{V_r}\times100$$

$$= \left(\frac{I_r R}{V_r}\cos\varphi_r + \frac{I_r X}{V_r}\sin\varphi_r\right)\times100$$

$$\text{regulation per unit} = \frac{I_r R}{V_r}\cos\varphi_r + \frac{I_r X}{V_r}\sin\varphi_r$$

$$= v_r \cos\varphi_r + v_x \sin\varphi_r$$

v_r and v_x are the per unit values of resistance and reactance of the line.

Constants of the network dimensionaless

The constants A, B, C and D are related for a passive network as follows

$$V_s = V_r + I_r Z$$

$$I_s = I_r$$

$$A = 1$$

$$B = Z$$

$$C = 0$$

$$D = 1$$

1.1 – Z.0 = 1

1.2 $V_r' = \frac{V_s}{A}$ when $I_r = 0$

1.3 $\%\text{regulation} = \frac{V_s / A - V_r}{V_r}\times100$

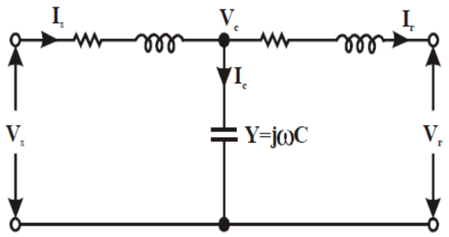
% η of transmission

$$\%\eta = \frac{\text{Power received at the receiving end}}{\text{Power received at the receiving end} + \text{losses}} \times 100$$

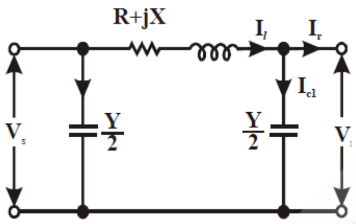
$$= \frac{P}{P + 3I_r^2 R} \times 100$$

R is the resistance per phase of the line

1.9.3 MEDIUM LENGTH LINES



(a) Nominal-T



(b)Nominal-π

The parameters are assumed to be lumped. The shunt capacitance at the middle at each end. Noted that two representations are approximate to the exact representation of the actual line

Nominal-T

For lagging power factor load receiving end current reference vector.

To calculate regulation

$$V_r' = \frac{|V_s|\left(-\frac{j}{\omega C}\right)}{\frac{R}{2} + j\frac{X}{2} - \frac{j}{\omega C}}$$

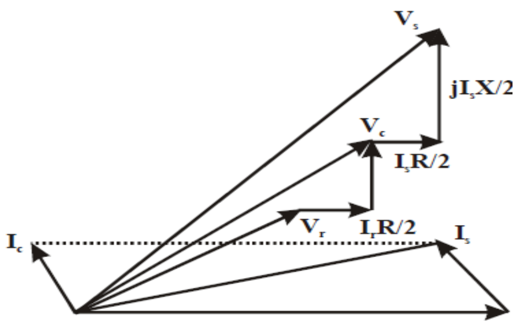


Fig. Phasor diagram for nominal-T

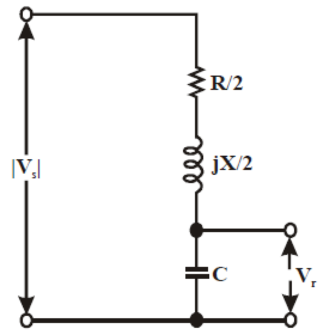


Fig. Equivalent circuit under no load

An small error in evaluating phase angle inaccurate calculation of efficiency

$$\%\eta = \frac{P}{P + 3\frac{R}{2}(I_r^2 + I_s^2)} \times 100$$

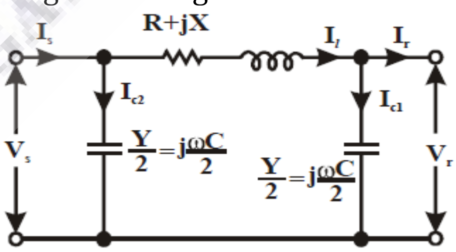
$$V_s = V_r\left(1 + \frac{YZ}{2}\right) + I_r Z\left(1 + \frac{YZ}{4}\right)$$

$$I_s = I_r\left(1 + \frac{YZ}{2}\right) + V_r Y$$

$$= YV_r + \left(1 + \frac{YZ}{2}\right)I_r \quad A = D \text{ and } AD - BC = 1$$

Nominal-π

Receiving end voltage as the reference or



(a)Nominal-π

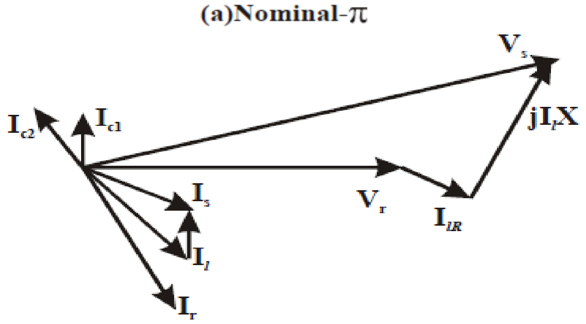


Fig. (b) Phasor diaram for nominal-π

$$V_r' = \frac{|V_s|\left(-\frac{2j}{\omega C}\right)}{R + jX - \frac{j}{\omega C/2}}$$

$$\%\eta = \frac{P}{P + 3I_l^2 R} \times 100$$

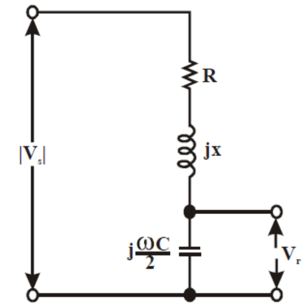


Fig. Equivalent circuit under no load

$$V_s = V_r' + I_l Z = V_r + \left(I_r + V_r \frac{Y}{2} \right) Z$$

$$= \left(1 + \frac{YZ}{2} \right) V_r + Z I_r$$

$$= \left(1 + \frac{Y^2 Z}{4} \right) + \left(1 + \frac{YZ}{2} \right) I_r$$

$$A = D$$

1.9.3 LONG TRANSMISSION LINES

The receiving end as the reference for measuring distances

When $\Delta x \to 0$ reduce to

$$V = A \exp\left(\sqrt{yz}.x \right) + B \exp\left(-\sqrt{yz}.x \right)$$

$$Z_c = \sqrt{\frac{z}{y}} \text{and} \gamma = \sqrt{yz} = \alpha + j\beta$$

Z_c characteristic impedance and γ the propagation constant

$$V = Ae^{\gamma x} + Be^{-\gamma x}$$

$$1 = \frac{1}{Z_c}(Ae^{yx} - Be^{-yx})$$

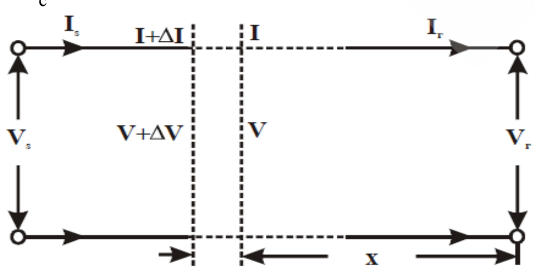


Fig. Long transmission line

Two constants are to be determined, two boundary conditions

At x = 0

$$A = \frac{V_r + I_r + Z_c}{2} \text{and } B = \frac{V_r - I_r Z_c}{2}$$

V and I (magnitude and phase) are functions of the distance x, receiving end voltage V_r and current I_r and the parameters of the line.

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L}{g + j\omega C}}$$

For a lossless line $r = 0, \quad g = 0,$

$$Z_c = \sqrt{\frac{L}{C}}$$

Pure resistance surge impedance of the line. When dealing with frequencies or surges normally the losses are neglected and, therefore, the characteristic impedance becomes the surge impedance. Surge impedance loading of a line is the power about 40 ohms. The phase angle of Z_c between 0^o and -15^oA. Line of infinite length cannot have a reflected wave.

Lower value cable, relatively large and low inductances of the cables

The propagation constant

$$V = \frac{V_r + I_r Z_c}{2} e^{ax}.e^{j\beta x} + \frac{V_r - I_r Z_c}{2} e^{-ax}.e^{-j\beta x}$$

The first term the incident voltage wave as Its value increases as x is increased. A voltage wave decreases in magnitude as it travels from the sending end towards the receiving end.

Second part reflected voltage. At any point along the line, voltage is the sum of these two components i.e.

sums of incident and reflected voltages

$$V = V_r.\frac{e^{\gamma x} + e^{-\gamma x}}{2} + I_r Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$= V_r \cosh \gamma x + I_l Z_c \sinh \gamma x$$

$$= \frac{1}{Z_c}[V_r \sinh \gamma x + I_r Z_c \cosh \gamma x]$$

$$\frac{V_r}{Z_c} \sinh \gamma x + I_r \cosh \gamma x$$

$$V_s = V_r \cos \gamma x + I_r Z_c \sin \gamma l$$

$A = D = \cos \ \gamma l$

ABCD Constants

$$V_s = AV_r + BI_r$$

$I_s = CV_r + DI_r$

Similar relations for V_r and I_r

Since AD − BC = 1 and A = D,

$I_r - CV_s + DI_s$

$V_r = DV_s - BI_s = AV_s - BI_s$

1.10 CONSTANTS FOR TWO NETWORKS IN TANDEM

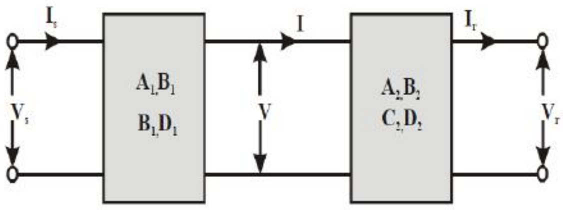


Fig. Two networks in tandem

$V = D_1V_s - B_1I_s$

$I = -C_1V_s + A_1I_s$

$V = A_2V_r + B_2I_r$

$I = C_2V_r + D_2I_r$

$D_1V_s - B_1I_s = A_2V_r + B_2I_r$

$-C_1V_s + A_1I_s = C_2V_s + D_2I_r$

Multiplying by A_1 and by B_1 and adding

$(A_1D_1 - B_1C_1)I_s = (A_2C_1 + C_2D_1)$

$V_r + (B_2C_1 + D_1D_2)I_r$

$A = A_1A_2 + B_1C_2$

$B = A_1B_2 + B_1D_2$

$C = A_2C_1 + C_2D_1$

$D = B_2C_1 + D_1D_2$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

1.11 CONSTANTS FOR TWO NETWORKS IN PARALLEL

The derivation is based on the fact that transmission line is a reciprocal network (symmetrical network) and when two reciprocal networks are connected in parallel, the resulting network is also reciprocal (The resulting networks is not reciprocal in case the two networks are connected in tandem).

$V_s = A_1V_r + B_1I_{r_1}$

$V_s = A_2V_r + B_2I_{r_2}$

Where $I_r = I_{r_1} + I_{r_2}$

Multiplying by B_2 and B_1

$(B_1 + B_2)V_s = (A_1B_2 + A_2B_1)V_r + B_1B_2(I_{r_1} + I_{r_2})$

$$V_s = \frac{A_1B_2 + A_2B_1}{B_1 + B_2}V_r + \frac{B_1B_2}{B_1 + B_2}I_r$$

$$A = D = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} = \frac{D_1B_2 + D_2B_1}{B_1 + B_2}$$

$AD - BC = 1$

Measured by conducting the open circuit and short circuit tests at the two ends of the line

$$Z_{so} = \frac{A}{C}$$

$$Z_{ss} = \frac{B}{D}$$

Since the direction of sending end current according to the above equation enters the network whereas while performing the tests on receiving end side, the direction of the current will be leaving the network, therefore

$I_{r_1} = DV_s + BI_s$

$-I_r = -CV_s - AI_s \text{ or } I_r = CV_s + AI_s]$

$$Z_{ro} = \frac{V_r}{I_r} = \frac{D}{C}$$

$$Z_{rs} = \frac{V_r}{I_r} = \frac{B}{A}$$

$$A = \sqrt{\frac{Z_{so}}{Z_{ro} - Z_{rs}}}$$

$B = AZ_{rs}$

$$C = \frac{A}{Z_{so}}$$

$D = A$

1.12 FERRANTI-EFFECT

When a long line is operating under no load or light load condition, the receiving end voltage is greater than the sending end voltage.

Ferranti-effect

i) No load condition
When x = l and $I_r = 0$
At $l = 0$

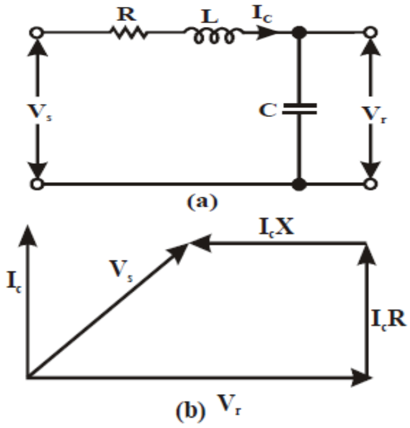


Fig. (a) Line representation (Lumped) under no load condition (b) its phasor diagram

As l increases, the incident component of sending end voltage increases and turns the vector anti-clockwise through an angle βl whereas the reflected part. Rotated clockwise through the same angle.
Sum sending end voltage gives a voltage which is smaller than V_r

ii) A simple explanation of Ferranti-effect
Since usually the capacitive reactance of the line is quite large as compared to the inductive reactance, under no load or lightly loaded condition the line current is of leading p.f.
The phasor diagram is given below for this operating condition
The charging current produces drop in the reactance of the line which is in phase opposition to the receiving end voltage and hence the sending end voltage becomes smaller than the receiving end voltage.

The Ferranti-effect is based on the net reactive power flow on the line. It is known that if the reactive power generated at a point is more than the reactive power absorbed, the voltage at that point becomes higher than the normal value and vice versa. The inductive reactance of the line is a sink for the reactive power whereas the shunt capacitances generate reactive power. In fact, if the line loading corresponds to the surge impedance loading, the voltage is same everywhere as the reactive power absorbed then the equals the reactive power generated by the line. The SIL lightly loaded or fully loaded lines. If the loading is less than SIL, the reactive power generated is more than absorbed, therefore, the receiving end voltage is greater than the sending end voltage.

GATE QUESTIONS

Topics **Page No**

1 PERFORMANCE OF TRANSMISSION LINES, LINE PARAMETERS & CORONA

Q.1 The conductors of a 10km long. Single phase, two wire line are separated by a distance of 1.5m. The diameter of each conductor is 1cm. if the conductor are of copper,the inductance of the circuit is

a)50.0mH b)45.3mH
c)23.8mH d)19.6mH

[GATE-2001]

Q.2 Consider a long, two –wire line composed of solid conductors. The radius of both conductors is 0.25 cm and the distance between their centers is / meters. If this distance is doubled, then the inductance per unit length

a)Doubles
b) Halves
c) Increase but does not double
d) Decrease but does not halve

[GATE-2002]

Q.3 Bundle conductors are mainly used in high voltage overhead transmission lines to

a) Reduce transmission line losses
b) Increases mechanical strength of the line
c) Reduce corona
d) Reduce sag

[GATE-2003]

Q.4 The ABCD parameters of a 3-phase overhead transmission line are $A = D = 0.9\angle 0 B = 200\angle 90°\Omega$ and $C = 0.95 \times 10^3 \angle 90°$ S. At no-load condition a shut inductive reactor is connected at the receiving end of the line to limit the receiving –end voltage to be equal to the sending –end voltage. The ohmic value of the reactor is

a)∞ Ω b)2000Ω
c) 105.26Ω d) 1052.6Ω

[GATE-2003]

Q.5 A surge of 20kV magnitude travels along a lossless cable towards its junction with two identical lossless overheads transmission lines. The inductance and the capacitance of the cable are 0.4mH and 0.5μF per km. The inductance and capacitance of the overhead transmission lines are 1.5 mH and 0.015 μF per km. The magnitude of the voltage at the junction due to surge is

a)36.72kV b)18.36kV
c)6.07kV d)33.93kV

[GATE-2003]

Q.6 The generalized circuit constants of a 3-phase, 220 kV rated voltage, medium length transmission line are

$A=D=0.936+j0.016=0.936\angle 0.98°$
$B = 33.5 + j138 = 142.0\angle 76.4°\Omega$
$C = (-5.18 + j914) \times 10^{-6}℧$

If the load at the receiving end is 50 MW at 220 kV with a power factor of 0.9 lagging then magnitude of line to line sending end voltage should be

a)133.23kV b)220.00kV
c)230.78kV d)246. 30kV

[GATE-2004]

Q.7 The parameters of transposed overhead transmission line are given as: Self reactance $X_s = 0.4\Omega/km$ and Mutual reactance $X_m = 0.1\Omega/km$

The positive sequence reactance X_1 and zero sequence reactance X_0 respectively, in Ω/km are

a) 0.3,0.2 b)0.5,02
c)0.5,0.6 d)0.3,0.6

[GATE-2005]

Q.8 The concept of an electrically short, medium and long lie is primarily based on the

EXPLANATIONS

Q.1 (c)

$$L_{ab}=0.4\ln\left(\frac{d}{r'}\right)mH/km$$

$$=0.4\ln\left(\frac{150}{0.7788\times\frac{1}{2}}\right)mH/km$$

$=2.3815mH/km$

For 10kmlong line

$L_{ab}=23.815mH/km$

Q.2 (c)

$$L\propto\ln\left(\frac{d}{r'}\right)$$

$\ln(2d)>\ln d$

Q.3 (c)

- By bonding of conductors the self GMD of the conductors is increased.
- Corona loss $\propto(V-V_o)^2$

  V_o is approximately directly proportional to the size of the conductor, hence larger the size of the conductor larger will be the critical disruptive voltage and smaller will be the factor $(V-V_o)^2$ and hence, smaller will be the corona loss.

Q.4 (b)

At no-load condition

Active power at receiving end $=P_R=0$

Reactive power at receiving end $=Q_R$

=reactive power absorbed by the reactor to make $|V_S|=|V_R|$

$A=D=0.9\angle0°$

$|A|=0.9$ and $\alpha=0°$

$B=200\angle90°\Omega$

$|B|=200$ and $\beta=90°$

$$S_R=\frac{|V_S||V_R|}{|B|}\angle(\beta-\delta)-\left|\frac{A}{B}\right||V_R|^2\angle(\beta-\alpha)$$

The above equation is expressed in real and imaginary parts, we can write the real and imaginary parts, we can write the real and reactive powers at the receiving –end as

$$P_R=\frac{|V_S||V_R|}{|B|}\cos(\beta-\delta)-\left|\frac{A}{B}\right||V_R|^2\cos(\beta-\alpha)$$

...(i)

$$Q_R=\frac{|V_S||V_R|}{|B|}\text{sinsinc}(\beta-\delta)-\left|\frac{A}{B}\right||V_R|^2\sin(\beta-\alpha)$$

...(ii)

Since, $P_R=0$, equation (i) becomes

$$0=\frac{|V_R|^2}{200}\cos(90°-\delta)-\frac{0.9}{200}|V_R|^2\cos(90°-0)0$$

$$=\frac{|V_R|^2}{200}\sin\delta$$

$\delta=0°$

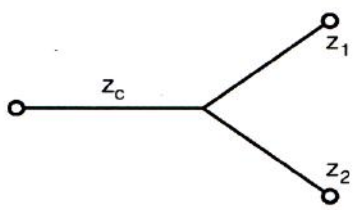
Equation (ii) becomes

$$Q_R=\frac{|V_R|^2}{200}\sin(90°-0)-\frac{0.9}{200}|V_R|^2\sin(90°-0)$$

$$Q_R=\frac{0.1}{200}|V_R|^2 \quad ....(iii)$$

Reactive power absorbed by rector of reactance x

Q.5 (d)

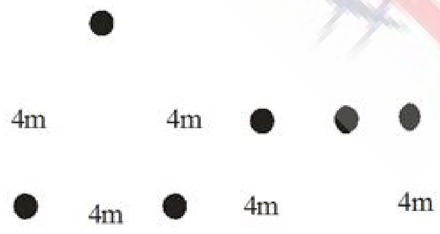


ASSIGNMENT QUESTIONS

Q.1 A medium line with parameters A, B, C, D is extended connecting a short line of impedance Z in series. The overall ABCD parameters of the series combination will be
a) A, AZ, C + D/Z, D
b) A, AZ + BC, CZ + D
c) A + BZ, B, C + DZ, D
d) AZ, B, C/Z, D

Q.2 The self-inductance of a long cylindrical conductor due to its internal flux linkages is kH/m. If the diameter of the conductor is doubled, then the self-inductance of the conductor due its internal flux linkages would be
a) 0.5 KH/m b) KH/m
c) 1.414 KH/m d) 4KH/m

Q.3 Two arrangements of conductors are proposed for a 3-phase transmission line : one with equilateral spacing of 4 m and the other, a flat with 4 m between the conductors as shown in the given figure.



The conductor diameter in each case is 2 cm. Assuming that the line is transposed in both cases. Which one of the following statements would be true?
(C_n = capacitance in F/m line to neutral, L = inductance in H/m per phase)
a) $C_{n1} = C_{n2}$ and $L_1 > L_2$
b) $C_{n1} > C_{n2}$ and $L_1 < L_2$
c) $C_{n1} < C_{n2}$ and $L_1 > L_2$
d) $C_{n1} > C_{n2}$ and $L_1 = L_2$

Q.4 When bundle conductors are used in place of single conductors, the effective inductance and capacitance will respectively
a) Increase and decrease
b) Decrease and increase
c) Decrease and remain unaffected
d) Remain unaffected and increase

Q.5 The inductance of a three-phase transmission line is 1.2 mH/phase/km. If the spacing of conductors and the radius of the conductor are doubled, then the inductance of the line will be
a) 4.8 mH/phase/km
b) 1.2 mH/phase/km
c) (In2) ×1.2 mH/phase/km
d) (1n 4) × 1.2 mH/phase/km

Q.6 The per-unit "regulation" of a short transmission line, having a per-unit resistance voltage drop v_r and a per-unit reactance voltage drop of v_x at rated current and, with power-factor cosθ, is given by
a) $(v_r + v_x)\cos\theta$ b) $(v_r + v_x)\sin\theta$
c) $v_r\cos\theta + v_x\sin\theta$ d) $v_r\sin\theta + v_x\cos\theta$

Q.7 The surge impedance of a 100 km long underground cable is 50 ohms. The surge impedance of a 40 km long similar cable would be
a) 20 ohms b) 50 ohms
c) 80 ohms d) 125 ohms

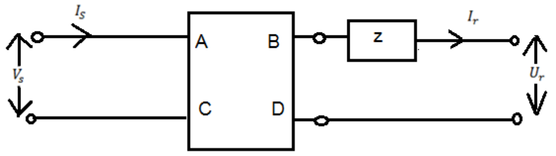
Q.8 A 3-phase overhead transmission line has its conductors horizontally spaced with spacing between adjacent conductors equal to \`d\`. If, now, the conductors of the lines are rearranged to form an equilateral of sides equal to \`d\`, then the

EXPLANATIONS

Q.1 (b)

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} A & AZ+B \\ C & CZ+D \end{bmatrix}$$



Q.2 (b)

Lint = KH/m

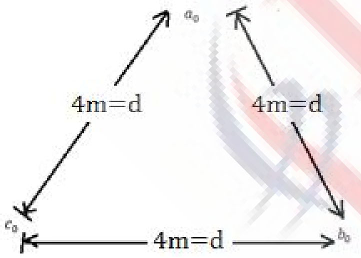
If the diameter of the conductor is doubled

$$\text{Lint} = \frac{\mu_o \mu_r}{8\pi} \rightarrow \text{Independent of length}$$

and diameter of the conductor

Lint = KH/m

Q.3 (c)



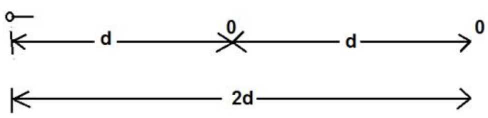
$$GMD = \sqrt[3]{d.d.d}$$

$= d$

GMR = selfGMD = r'

$L/ph \propto GMD$

$$C/ph \propto \frac{1}{GMD}$$



$$GMD_1 = \sqrt[3]{d.d.(2d)}$$

$$= d.2^{1/3}$$

$GMD_1 > GMD$

$$L_1/ph = 2\times10^{-7}\ln\left(\frac{GMD}{GMR}\right)$$

$$C_1/ph = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{GMD_0}{GMR}\right)}$$

$C_{n1} < C_{n2}$

$L_2 > L_1$

Q.4 (a)

Because of Bundle conductor

Self GMD(= GMR) will increase

$$C/ph = \frac{2\pi\epsilon_0}{\ln\left(\frac{GMD}{GMR}\right)}$$ will increase

$$L/ph = 2\times10^{-7}\ln\left(\frac{GMD}{GMR}\right)$$ will decrease

C/ph ↑ and L/ph ↓

Q.5 (b)

$L = 1.2\text{mh / phase}$

$$= 2\times10^{-7}\ln\left(\frac{GMD}{GMR}\right) \quad \text{or}$$

$$= 2\times10^{-7}\ln\left(\frac{d}{r'}\right)$$

$d_1 = d \Rightarrow d_2 = 2d$

$r'_1 = 0.7788r \Rightarrow r'_2 = 0.7788\times2r$

$$L_2 = 2\times10^{-7}\ln\left(\frac{d_2}{r'_2}\right)$$

$$= 2\times10^{-7}\ln\left(\frac{d}{r'}\right)$$

= 1.2mH/phase

Q.6 (c)

$$\%\text{Regulation} = \frac{IR\cos\phi_r \pm I\,x\,\text{Sin}\phi_r}{V_R}$$

$$= \frac{I_R}{V_R}\cos\Phi_r \pm \frac{I_x}{V_R}\sin\phi_r$$

$$\frac{I_R}{V_R} = V_r = \text{per unit resistance drop}$$