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IES | GATE | PSU

MEASUREMENTS

For

ELECTRICAL ENGINEERING

INSTRUMENTATION ENGINEERING

MEASUREMENTS

SYLLABUS

Measurement concept, Classification of Measurement, Types of errors & Standard Measurement technics Analog Circuits, Measurement of Resistance, Inductance, Capacitance, Bridge Measurement, Concept of Cathode Ray Oscilloscope, CRO, Volt meter & Frequency measurement,

ANALYSIS OF GATE PAPERS

Electrical Engineering				Instrumentation Engineering		
Exam Year	**1 Mark Ques.**	**2 Mark Ques.**	**Total**	**1 Mark Ques.**	**2 Mark Ques.**	**Total**
2003	3	8	19	5	4	13
2004	3	7	17	5	9	23
2005	3	5	13	3	7	17
2006	2	4	10	2	5	12
2007	1	1	3	3	4	11
2008	1	2	5	2	6	14
2009	2	2	6	3	3	9
2010	2	1	4	4	4	12
2011	3	1	5	-	3	6
2012	3	1	5	3	1	5
2013	2	1	4	-	2	4
2014 Set-1	2	2	6	1	1	3
2014 Set-2	2	2	6			
2014 Set-3	2	2	6			
2015 Set-1	2	1	4	2	2	6
2015 Set-2	3	7	17			
2016 Set-1	0	0	0	4	3	10
2016 Set-2	1	2	5			
2017 Set-1	2	2	6	2	4	10
2017 Set-2	2	1	4			

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1 CHARACTERISTICS, ERRORS & STANDARDS

1.1 MEASUREMENTS

The measurement of a given quantity is essentially an act or the result of comparison between the quantity and a predefined standard. Since two quantities are compared, the result is expressed in numerical values.

1.1.1 METHODS OF MEASUREMENTS.

i) Direct Methods and
ii) Indirect Methods

Direct Methods:- In these methods, the unknown quantity is directly compared against a standard. The result is expressed as a numerical number and a unit Direct methods are quite common for the measurement of physical quantities like length, mass and time.

Indirect Methods:- Measurement by direct methods are not always possible, feasible and practicable. Then measurement is done by measuring Instruments.

Instruments and Measurement Systems:- Measurements involve the use of instruments as a physical means of determining quantities or variables. The earliest scientific instruments used the same three essential elements as our modern instruments do. These elements are:

i) a detector
ii) an intermediate transfer device
iii) an indicator, recorder or a storage device.

The history of development of instruments encompasses three phases of instruments, vis.:

i) mechanical instruments
ii) electrical instruments
iii) electronic instruments.

Summarizing, it may be stated that in general electronic instruments have

i) a higher sensitivity
ii) a faster response
iii) a greater flexibility
iv) lower weight
v) lower power consumption
vi) a higher degree of reliability than their mechanical or purely electrical counterparts.

1.2 CLASSIFICATION OF INSTRUMENTS

i) Absolute Instruments
ii) Secondary Instruments.

1. **Absolute Instruments.** These instruments give the magnitude of the quantity under measurement in terms of physical constants of the instrument. The examples of this class of instruments are Tangent Galvanometer and Rayleigh's Current Balance.
2. **Secondary Instruments.** These instruments are so constructed that the quantity being measured can only be measured by observing the output indicated by the instrument. These instruments are calibrated by comparison with an absolute instrument or another secondary instrument which has already been calibrated against an absolute instrument.

1.2.1 DEFLECTION AND NULL TYPE INSTRUMENTS.

Deflection Type:- The instruments of this type, the deflection of the instrument provides a basis for determining the quantity under measurement. The measured quantity produces some physical effect with deflects or produces a mechanical displacement of the moving system of the instrument.

NULL TYPE:- In a null type of instrument, a zero or null indication leads to determination of the magnitude of measured quantity. The null condition is dependent upon some other known conditions.

Comparison of Deflection and Null Type Instruments

i) Null type of instruments are more accurate than deflection type instruments.
ii) Null type instruments can be highly sensitive as compared with deflection type instruments
iii) Deflection type of instruments are more suited for measurements under dynamic conditions than null type of instruments whose intrinsic response is slower.

Applications of Measurement systems.

i) Monitoring of processes and operations,
ii) Control of processes and operations, and
iii) Experimental Engineering analysis.

Elements of a Generalized Measurement System.

1. Primary sensing element,
2. Variable conversion element,
3. Data presentation element.

Primary Sensing Element: A transducer is defined as a device which converts a physical quantity into an electrical quantity.

Variable Conversion Element: The output of the primary sensing element may be electrical signal of any form. It may be necessary to convert this output to some other suitable form while preserving the information content of the original signal.

Data Presentation Element: The information about the quantity under measurement has to be conveyed to the personnel handling the instrument or the system for monitoring, control, or analysis purposes. The information conveyed must be in a form intelligible to the personnel or to the intelligent instrumentation system.

Characteristics of Instruments and Measurement Systems

(i) Static characteristics, and
(ii) Dynamic characteristics.

Static Characteristics. The main static characteristics discussed here are:

i) Accuracy
ii) Sensitivity
iii) Reproducibility
iv) Drift
v) Static error
vi) Dead Zone

The qualities (i), (ii) and (iii) are desirable, while qualities (iv), (v) and (vi) are undesirable.

Static Error: The most important characteristic of an instrument of measurement system is its accuracy, the accuracy of an instrument is measured in terms of its error.

Static error is defined as the difference between the measured value and the true value of the quantity. Then:

$\delta A = A_m - A_t$

the ratio of absolute static error δA to A_t, the true value of the quantity under measurement. Therefore, the relative static error $\in_r$, is given by:

$$\in_r = \frac{\text{absolute error}}{\text{true value}} = \frac{\delta A}{A_t} = \frac{\in_0}{A_t}$$

$A_t = A_m (1 - \in_r)$

Accuracy: It is the closeness with which an instrument reading approaches the true value of the quantity being measured. Thus accuracy of a measurement means conformity to truth.

Precision: It is a measure of the reproducibility of the measurements, the term 'Precise' means clearly or sharply defined.

A Wheatstone bridge requires a change of 7 Ω in the unknown arm of the bridge to produce a change in deflection of 3mm of the galvanometer.

Sensitivity= $\dfrac{\text{magnitude of output response}}{\text{magnitude of input}}$

= $\dfrac{3\text{mm}}{7\Omega}$ = 0.429 mm/Ω

Inverse sensitivity or scale factor = $\dfrac{\text{magnitude of input}}{\text{magnitude of output response}}$ = $\dfrac{7\Omega}{3\text{mm}}$ = 2.33 Ω/mm

Linearity: One of the best characteristics of an instrument or a measurement system is considered to be linearity, that is, the output is linearly proportional to the input.

Dead Time: Dead time is defined as the time required by a measurement system to begin to respond to a change in the measured.

Dead Zone: It is defined as the largest change of input quantity for which there is no output of the instrument

Resolution or Discrimination: If the input is slowly increased from some arbitrary input value, it will again be found that output does not change at all until a certain increment is exceeded. This increment is called resolution or discrimination of the instrument. So resolution defines the smallest measurable input change.

Example

A moving coil voltmeter has a uniform scale with 100 divisions, the full scale reading is 200 V and 1/10 of a scale division can be estimated with a fair degree of certainty. Determine the resolution of the instrument in volt.

Solution

1 scale division = 200/100 = 2V

Resolution = $\dfrac{1}{10}$ scale division = $\dfrac{1}{10}\times 2$

=0.2 V

Example A digital voltmeter has a read-out range from 0 to 9,999 counts. Determine the resolution of the instrument in volt when the full scale reading is 9.999 V.

Solution. The resolution of this instrument is 1 or 1 count in 9,999.

$\therefore$ Resolution = $\dfrac{1}{9999}$ count = $\dfrac{1}{9999}\times$ 9.999 volt = 10^{-3} V = 1 mV.

Loading Effects: The ideal situation in a measurement system is that when an element used for any purpose may be for signal sensing, conditioning, transmission or detection is introduced into the system, the original signal should remain un-distorted. This means that the original signal should not be distorted in any form by introduction of any element in the measurement system. However, under practical conditions in extraction of energy from the system thereby distorting the original signal. This distortion may take the form of attenuation waveform distortion, phase shift and many a time all these undesirable features put together.

Errors in Measurements and Their Statistical Analysis

Actual value of quantity $A_a = A_s \pm \delta A$

Relative limiting error $\varepsilon_r = \dfrac{\delta A}{A_s} = \varepsilon_r A_s$

$$\varepsilon_r = \frac{\text{actual value} - \text{nominal value}}{\text{nominal value}}$$

Combination of Quantities with Limiting Errors.

i) Sum of two quantities. $X = x_1 + x_2$

$$\frac{dx}{X} = \frac{dx_1}{X} + \frac{dx_2}{X} = \frac{x_1}{X}\frac{dx_1}{x_1} + \frac{x_2}{X}\frac{dx_2}{x_2}$$

$$\frac{\delta X}{X} = \pm\left[\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right]$$

ii) Difference of two quantities.

$X = x_1 - x_2$

$$\therefore \quad \frac{\delta X}{X} = \frac{dx_1}{X} - \frac{dx_2}{X},$$

$$\frac{dX}{X} = \frac{dX}{X}\frac{dx_1}{x_1} - \frac{x_2}{X}\frac{dx_2}{x_2},$$

$$\frac{\delta X}{X} = \pm\left(\frac{x_1}{X}.\frac{\delta x_1}{x_1} + \frac{x_2}{X}.\frac{\delta x_2}{x_2}\right)$$

iii) Sum of difference of more than two quantities.
$X = \pm x_1 \pm x_2 \pm x_3$. then the relative limiting error in X is given by:

$$\frac{\delta X}{X} = \pm\left(\frac{x_1}{X}.\frac{\delta x_1}{x_1} + \frac{x_2}{X}.\frac{\delta x_2}{x_2} + \frac{x_3}{X}.\frac{\delta x_3}{x_3}\right)$$

iv) Product of two Components,
$X = x_1 x_2$.,
$\log_e X = \log_e x_1 + \log_e x_2$.

$$\frac{1}{X} = \frac{1}{x_1}.\frac{dx_1}{dX} + \frac{1}{x_2}.\frac{dx_2}{dX},$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2},$$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right)$$

v) Quotient
$X = \frac{x_1}{x_2}$,
$\log_e X = \log_e x_1 - \log_e x_2$.

$$\frac{1}{X} = \frac{1}{x_1}.\frac{dx_1}{dX} - \frac{1}{x_2}.\frac{dx_2}{dX},$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} - \frac{dx_2}{x_2},$$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right)$$

vi) Product or quotient of more than two quantities

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3}\right)$$

vii) Composite factors
$X = x_1^n . x_2^m$,
$\log_e X = n \log_e x_1 + m \log_e x_2$

$$\frac{1}{X} = \frac{n}{x_1}.\frac{dx_1}{dX} - \frac{m}{x_2}.\frac{dx_2}{dX},$$

$$\frac{dX}{X} = n\frac{dx_1}{x_1} + m\frac{dx_2}{x_2},$$

$$\frac{\delta X}{X} = \pm\left(n\frac{\delta x_1}{x_1} + m\frac{\delta x_2}{x_2}\right)$$

1.3 TYPES OF ERRORS

1. Gross Errors,
2. Systematic Error.
3. Random Errors.

Gross Errors. This class of errors mainly covers human mistakes in reading instruments and recording and calculation measurement results.
1. Great care should be taken in reading and recording the data.
2. Two, three or even more reading should be taken for the quantity under measurement.

Systematic Errors.
1. Instrumental Errors.
2. Environmental Errors.
3. Observational Errors.

1. Instrumental Errors.
   i) Due to inherent shortcomings in the instrument,
   ii) Due to misuse of the instruments, and
   iii) Due to loading effects of instruments

Environmental Errors.
Observational Errors.
Random Errors.
Statistical Treatment of Data.

i) Multi-sample test and
ii) Single-sample test.

Arithmetic Mean.

$$\overline{X} = \frac{x_1 + x_2 + x_3 + x_4 + \dots\dots\dots + x_n}{n} = \frac{\sum x}{n}$$

$d_1 = x_1 - \overline{X}$

$d_2 = x_2 - \overline{X}$
…………
$d_n = x_n - \overline{X}$

$$\overline{X} = \frac{\sum(x_n - d_n)}{n}$$

Average Deviation.

$$\overline{D} = \frac{|-d_1| + |-d_2| + |-d_3| + \ldots + |-d_n|}{n} = \frac{\sum|d|}{n}$$

Standard Deviation (S.D.)

$$S.D. = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}} = \sqrt{\frac{\sum d^2}{n}}$$

When the number of observations is greater than 20, S.D. is denoted by symbol σ while if the number of observation is less than 20, the symbol used is s.

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n-1}}$$

$$\sqrt{\frac{\sum d^2}{n-1}}$$

Variance. The variance is the mean square deviation,

V = (Standard Deviation)^2

= (S.D.)^2 = σ^2

$$= \frac{d_1^2 d_2^2 + d_3^2 + \ldots + d_n^2}{n}$$

$$= \frac{\sum d^2}{n}, V = s^2 = \frac{\sum d^2}{n-1}$$

1.4 STANDARDS

i) International standards
ii) Primary standards
iii) Secondary standards
iv) Working standards

(i) International standards : Not available to everyone.
(ii) Pri. standards: National standards
(iii) Sec. standards: used in industrial labs.
(iv) Working standards: Used in general labs.

Standards of EMF:

'Weston' cell is used for primary and secondary standards of emf.

Pri. Standard Weston cell: saturated, normal, Weston cell is used asthe pri standard of emf..

- The potential of saturated weston cell. E = 1.01864 volts.
- It contains $CdSO_4$ crystal, Hg_2SO_4, (Cd + Hg) (Amalgum).

Note : $CdSO_4$ crystal is used in saturated Weston cell only.

- Variation in emf with temperature –40 μv/°C
- Variation in potential with time. -1 μV/Year
- The max. current from saturated weston cell is 100 μA.
- Internal resistance of sat. weston cell. 600-800Ω

Sec. Standard

- Unsaturated weston cell is used as sec. standards.
- The potential of unsaturated weston cell E=l.0180 to 1.0194V.
- It does not have $CdSO_4$ crystal.
- Porous plug is used to hold electrode in place.
- Variation in potential is -30μV to -50μV/year.

Laboratory standard of emf

- The zener diode circuit is used for laboratorystandard.

Standards of Resistance

Magnine is used for the standard resistance.
Contents of Magnine:
Ni → 4%
Cu → 84%
Mn → 12%

Characteristics of Magnine

- High resistivity

- low temp. coefff.
- low thermal expansion with copper.

Errors in resistance standards

- Skin effect.
- stray inductances, and capacitances.
- there can be contact resistances.

Bifilar winding

The bifilar winding is used to reduce the inductive effect of resistance.

Campbell type

is used as the primary standard. It consists of marble cylinders with screw threads carrying a coils of bare copper, Bare copper (without any insulation) wound under tension.

Sec, standards of mutual inductance

It consists of two coils wound on bobbin of marble and coils are separated by a flange. Cu is used as a conductor.

Pri. standards of self inductance

It is same as that of mutual inductance.
(i.e. Campbell type).

Sec. standards of self inductance.

Silk covered copper wire wound on marble former.

Pri. standards of time

Atomic clock is used as primary standard.

Pri._standards of freq.

a) CAESIUM (Ce) beam is used as pri-standard
b) Hydrogen maser.

Sec. standards of freq.

a) rubidium crystal
b) Quartz crystal

GATE QUESTIONS

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CHARACTERISTICS OF INSTRUMENTS & MEASUREMENT SYSTEMS

Q.1 Resistance R_1 and R_2 have respectively, nominal value of 10Ω and 5Ω and tolerances of ±5% and ±10%. The range of values for the parallel combination of R_1 and R_2 is

a) 3.077Ω to3.616Ω
b) 2.805Ω to3.371Ω
c) 3.237Ω to3.678Ω
d) 3.192Ω to3.435Ω

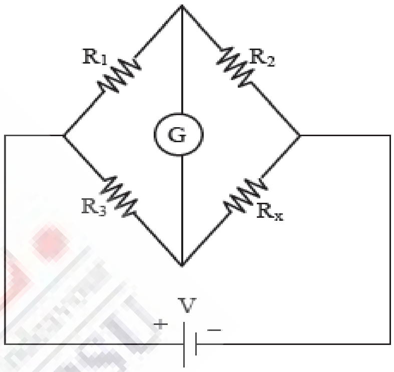
[GATE-2001]

Q.2 A variable w is related to three other variables x, y, z as w=xy/z. The variables are measured with meters of accuracy ±0.5% reading, ±1% of full scale value and ±1.5% reading. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. The maximum uncertainty in the measurement of w will be

a) ±0.5%rdg b) ±5.5%rdg
c) ±6.7%rdg d) ±7.0%rdg

[GATE-2006]

Q.3 When the Wheatstone bridge shown in the figure is used to find the value of resistor R_x, the galvanometer G indicates zero current when R_1=50 Ω, R_2 = 65 Ω and R_3 = 100 Ω, If R_3 is known with ± 5% tolerance on its nominal value of 100 Ω . What is the range of R_x in Ohms?



a) [123.50, 136.50]
b)[125.89, 134.12]
c) [117.00, 143.00]
d) [120.25, 139.75]

[GATE-15-1]

ANSWER KEY:

1	2	3
(a)	(d)	(a)

EXPLANATIONS

Q.1 (a)

Range of $R_1 = 10 \pm 10 \times \frac{5}{100}$

$= 9.5\Omega$ to 10.5Ω

Range of

$R_2 = 5 \pm 5 \times \frac{10}{100} = 4.5\Omega$ to 5.5Ω

$\because R_p = \frac{R_1 R_2}{R_1 + R_2}$

$\therefore$ Range of $R_p = \frac{9.5 \times 4.5}{9.5 + 4.5}$ to $\frac{10.5 \times 5.5}{10.5 + 5.5}$

$= 3.05\Omega$ to 3.61Ω

Q.2 (d)

Full scale reading of all three =100

Readings of x=80

Readings of y=20

Readings of z=50

$\delta x = \pm 0.5\%$ of reading $= \pm \frac{0.5 \times 80}{100} = \pm 0.4$

$\delta y = \pm 1\%$ of full reading $= \pm \frac{1 \times 100}{100} =$

$\pm 1 \delta z = \pm 1.5\%$ of reading $= \pm \frac{1.5 \times 50}{100}$

$= \pm 0.75$

Given $\omega = \frac{xy}{z}$

Taking log, we get

$\log\omega = \log x + \log y - \log z$

Differenting wrt ω we get

$\frac{\delta\omega}{\omega} = \frac{\delta x}{x} + \frac{\delta y}{y} - \frac{\delta z}{z}$

For maximum uncertainty

$\frac{\boldsymbol{\delta\omega}}{\boldsymbol{\omega}} = \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{100}\right) \times 100 = \pm 7\%$

Q.3 (a)

Weinbridge is balanced,

$R_1, R_x = R_2 R_3$

$50 \times R_x = 65 \times 100$

$R_x = 130\Omega$

Now $R_3 = 100 \pm 100 \times 0.05 = 100 \pm 5$

$= 95/105\ \Omega$

$R_x = \frac{R_2 R_3}{R_1} = \frac{65 \times 105}{50} = 136.5\Omega$

$R_x = \frac{65 \times 95}{50} = 123.5\Omega$

Range of R is123.5 Ω to136.5 Ω

ASSIGNMENT QUESTIONS

Q.1 A Wheatstone bridge requires a change of 6 ohms in the unknown arm of the brigeto produce a change in deflection of 3 mm of the galvanometer. The sensitivity of the instrument is

a) 0.5 percent b) 2.0 percent
c) 0.5 mm/ohm d)2.0ohm/mm

Q.2 A digital voltmeter has a read-out range from 0 to 0000 counts. When full scale reading is 0.999 V, the resolution of the full scale reading is

a) 0.001 b) 1000
c) 3 digit d) 1 mv

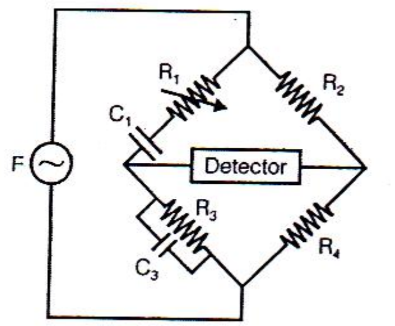
Q.3 The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. If the limiting errors in the measurement of power and current are ±1.5% and ±1.0%

a) ±1% b) ±1.5%
c) ±2.5% d) ±3.5%

Q.4 A simple dc potentiometer is to be standardized by keeping the slider wire setting at 1.0183 V. If by mistake, the setting is at 1.0138 V and the standardization is made to obtain a source voltage of 1.0138 V, then the reading of the potentiometer will be

a) 1.0138 V b) 1.0183 V
c) $\frac{(1.0138)^2}{1.0183}V$ d) $(1.0138)^2$ V

Q.5 Which of the following conditions are to be satisfied so that the common variableshaft of resistance R_1 and R_3 can be graduated in frequency to measure thefrequency of under balanced condition



a) $R_1 = R_3$ b) $C_1 = C_3$
c) $R_2 = 2 R_4$ d) $R_2 = R_4$

Q.6 Consider the following statements in connection with deflection and null type instruments:

1. Null type instrument is more is more accurate than the deflection type one.
2. Null type instrument can be highly sensitive as compared with deflection type instrument.
3. Under dynamic consideration, null type instrument is not proffered to deflection type instrument.
4. Response is faster in null type instrument as compared to deflection type instrument.

Which of these statements are correct?

a)1, 2 and 3 only b)1, 2 and 4 only
c)2, 3 and 4 only d)1, 2, 3 and 4.

Q.7 In the statement "the wattmeter commonly used to power measurement atcommercial frequencies is of the X-type. This meter consists of two coil systems, the fixed system being the Y-coil and moving system being the Z-coil". X, Y and Z stand respectively for

a) dynamometer, voltage & current
b) dynamometer, current & voltage
c) induction, voltage and current
d) induction, current and voltage

EXPLANATIONS

Q.1 (c)

Sensitivity = $\frac{3}{6}$ mm/ohm

Q.2 (d)

Resolution = $\frac{9.999}{9999}$ = 1mV

Q.3 (d)

$P = I^2R$

$R = \frac{P}{I^2}$

$\frac{\delta R}{R} = \frac{\delta P}{P} \pm \frac{2\delta I}{I}$

$= \pm 1.5\times(2\times1)$

$\frac{\delta R}{R} = \pm 3.5\%$

Q.4 (a)

Q.5 (d)

Q.6 (a)

Q.7 (b)

In dynamometer type wattmeter, the fixed coil is current coil and moving coil us voltage coil or pressure coil.

Q.8 (c)

When power factor $\cos\phi$ = 0.5 then one wattmeter reads zero. The load is lagging in this case.

Q.9 (a)

Q.10 (a)

Meter constt.=400 revolution parkwh

$= 400\times\frac{240\times10\times0.8}{1000}$ revolutions per hour

= 768 revolution per hour

$= \frac{768}{60}$ rpm = 12.8 rpm

Q.11 (c)

Q.12 (c)

Q.13 (b)

$\frac{fy}{fx} = \frac{\text{No. of horizontal tangent}}{\text{No. of vertical tangle}}$

$\frac{fy}{1000} = \frac{5}{2}$

fy = 2500 Hz

Q.14 (a)

Q.15 (d)

Q.16 (b)

$\frac{\delta V}{V} = 2\%$

$\delta V = \frac{2}{100}\times300 = 6$ V

So range of reading = 30 ± 6

= 24 V to 36

Q.17 (b)

Q.18 (b)

Q.19 (c)

Q.20 (a)

Q.21 (a)

Because PMMC reads only d.c. value or average value.

Q.22 (a)