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HEAT TRANSFER

For

MECHANICAL ENGINEERING

HEAT TRANSFER

SYLLABUS

Modes of heat transfer; one dimensional heat conduction, resistance concept and electrical analogy, heat transfer through fins; unsteady heat conduction, lumped parameter system, Heisler's charts; thermal boundary layer, dimensionless parameters in free and forced convective heat transfer, heat transfer correlations for flow over flat plates and through pipes, effect of turbulence; heat exchanger performance, LMTD and NTU methods; radiative heat transfer, Stefan-Boltzmann law, Wien's displacement law, black and grey surfaces, view factors, radiation network analysis.

ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	1	4	9
2004	1	3	7
2005	3	6	15
2006	1	2	5
2007	-	7	14
2008	1	4	9
2009	1	4	9
2010	-	2	4
2011	2	3	8
2012	3	4	11
2013	3	4	11
2014 Set-1	2	2	6
2014 Set-2	2	3	8
2014 Set-3	1	3	7
2014 Set-4	2	3	8
2015 Set-1	2	2	6
2015 Set-2	2	3	8
2015 Set-3	2	2	6
2016 Set-1	1	3	7
2016 Set-2	2	2	6
2016 Set-3	3	2	7
2017 Set-1	1	2	5
2017 Set-2	2	2	6

CONTENTS

1 INTRODUCTION TO MODES OF HEAT TRANSFER

From thermodynamics we learned that energy can be exchanged between systems and surrounding. This energy in transit is called work and heat. The thermodynamics deals with the end state of process during which interaction but does not explain the way heat of interactions. The objective of **heat transfer** is to extend thermodynamic analysis through study of the modes of heat transfer and development of relations to calculate heat transfer rates.

Heat is the energy in transit due to difference in temperature. Heat always flows from higher temperature to lower temperature. The various modes by which the heat transfer can take place are

1. Conduction
2. Convection
3. Radiation

1.1 CONDUCTION

Conduction is the transfer of energy form more energetic particle to adjacent less energetic particle. Conduction takes place in solid, liquid and gases. In solids conduction takes by transport of energy from higher temperature region to lower temperature region by free electrons or by vibration of molecules. In liquids and gases, the energy is transported due to collision or diffusion of molecules during their random motion. The rate of heat transfer by conduction can be determined by **Fourier's law** which states that

$$Q_{cond} = -kA\frac{\Delta T}{\Delta x}$$

For Lim $\Delta x \rightarrow 0$, equation can be written as

$$Q_{cond} = -kA\frac{dT}{dx}$$

1.2 CONVECTION

Convection heat transfer mainly occurs between a fluid in motion and a surface when the two are at different temperatures.The convection heat transfer mode is comprised of two mechanisms

1. Energy transfer due to random molecular motion (diffusion),
2. Energy transferred by the bulk or macroscopic, motion of the fluid.

This fluid motion is associated with the fact that, at any instant, large numbers of molecules are moving collectively as a bunch. Such motion, in the presence of a temperature difference leads to heat transfer. The molecules in the bunch also have random motion; the total heat transfer is then due to energy transport by the random motion of the molecules and by the bulk motion of the fluid

When surface comes in contact with moving fluid, a region is developed near to the surface in which velocity gradient exists. This region is called **hydrodynamic boundary layer**.similary, if the surface and flow temperatures differ, there will be a region of the fluid through which the temperature varies .This region, called the **thermal boundary layer**.Convection heat transfer may be classified according to the nature of the flow. **Forced convection** is when flow is caused by external means, such as by a fan, a pump, or atmospheric winds. **Free (or natural) convection**, the flow is induced by buoyancy forces, which are due to density differences caused by temperature variations in the fluid. An example is the free convection heat transfer that occurs from hot components on a vertical array of circuit boards in air

The heat transfer rate between fluid and surface is determined by Newton's Law of cooling

$$Q_{conv} = hA\left(T_S - T_\infty\right)$$

Where,

Q_{conv}	Convective heat flow rate, W
A	Surface in fluid contact, m^2

T_S Temp of surface, K

T_∞ Temp of fluid, K

The constant of proportionality 'h' is known as heat transfer coefficient. Convection heat transfer coefficient can be defined as the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temp difference. Its unit is W/m^2K.the value of heat transfer coefficient depends mainly on type of flow, surface condition, velocity of fluid and thermo physical properties of fluid.

Typical values of Heat transfer coefficient

1. Free convection
   Gases 2–25
   Liquids 50–1000
2. Forced convection
   Gases 25–250
   Liquids 100–20,000
3. Convection with phase change
   Boiling or condensation 2500–100,000

1.3 THERMAL RADIATIONS

Thermal radiation is energy emitted by matter that is at a nonzero temperature. Thermal radiation vane is emitted by solids, liquids and gases. Regardless of the form of matter, the emission is due changes in the electron configurations of the constituent atoms or molecules. The radiation energy is transported by electromagnetic waves (oralternatively, photons). While the transfer of energy by conduction or convectionrequires the presence of a material medium, radiation does not need any medium. Consider radiation transfer processes for the surface. Radiationthat is emitted by the surface originates from the thermal energy of matter boundedby the surface, and the rate at which energy is released per unit area (W/m^2) istermed the **surface emissive power (E)**. This is maximum emissive power a surface can emit at given temperature. Such surface emitting maximum emissive power is known as **black body**.

The radiations emitted by blackbody is given by the **Stefan–Boltzmann law**

$$E_b = \sigma\left(T_S^4\right)$$

Where, Stefan –Boltzmann constant $\sigma = 5.67\times10^{-8}\,w/m^2k^4$, T_s is surface temperature (K)

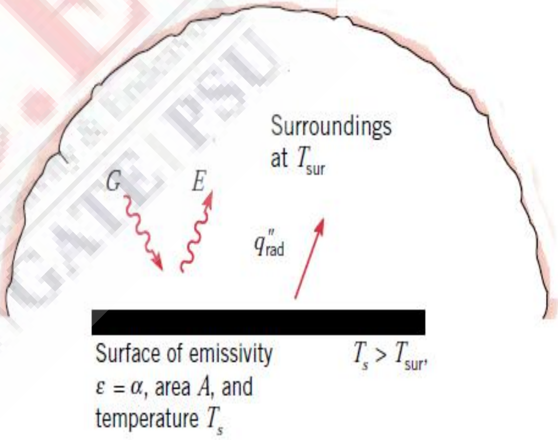
The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$E = \varepsilon\sigma\left(T_S^4\right)$$

Where , ε is emissivity

The heat interaction between surface and surrounding is shown in figure.

The rate at which all radiations is incident on a unit area of the surface is called irradiations G (W/m^2).



Net rate of heat transfer from surface is give by

Q = Toatal radiations emitted – Toatl radiations absorbed

$$Q = E - \alpha G = \varepsilon\sigma\left(T_S^4\right) - \alpha\sigma\left(T_S^4\right)$$

According to Kirchhoff's Law (ε=α), where, ε is emissivity &αis absorbity

$$Q = \varepsilon\sigma\left(T_S^4 - T_{sur}^4\right)$$

GATE QUESTIONS

Topics **Page No**

1 CONDUCTION

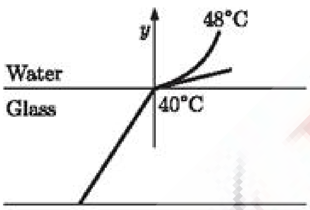
Q.1 In descending order of magnitude, the thermal conductivity of (a) pure iron, (b) liquid water, (c) saturated water vapour and (d) aluminum can be arranged as

a) abcd　　b) bcad
c) dabc　　d) dcba

[GATE – 2001]

Common Data for Q.2 and Q.3

Heat is being transferred by convection from water at 48°C to a glass plate whose surface that is exposed to the water is at 40°C. The thermal conductivity of water is 0.6 W/mK and the thermal conductivity of glass is 1.2 W/mK. The spatial gradient of temperature in the water at the water-glass interface is dT/dy 1 × 10^4 K/m.



Q.2 The value of the temperature gradient in the glass at the water-glass interface in K/m is

a) -2×10^4　　b) 0.0
c) 0.5×10^4　　d) 2×10^4

[GATE – 2003]

Q.3 The heat transfer coefficient h in W/m^{2}K is

a) 0.0　　b) 4.8
c) 6　　d) 750

[GATE – 2003]

Q.4 One dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of 'q ' can be written as

a) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

b) $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

c) $\frac{\partial^2 T}{\partial r^2}+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

d) $\frac{\partial^2}{\partial r^2}(rT)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

[GATE – 2004]

Q.5 A stainless steel tube (k_S 19 W/mK) of 2 cm ID and 5 cm OD is insulated with 3 cm thick asbestos (k_a 0.2 W/mK). If the temperature difference between the innermost and outermost surfaces is 600^0C, the heat transfer rate per unit length is

a) 0.94 W/m　　b)9.44W/m
c) 944.72 W/m　　d)9447.21W/m

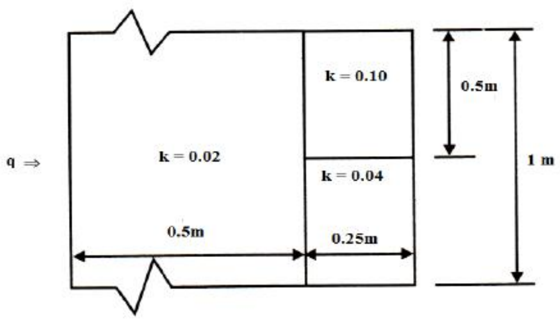
[GATE – 2004]

Q.6 In a case of one dimensional heat conduction in a medium with constant properties, T is the temperature at position x, at time t. Then $\frac{\partial T}{\partial t}$ is proportional to

a) $\frac{T}{x}$　　b) $\frac{\partial T}{\partial x}$
c) $\frac{\partial^2 T}{\partial x \partial t}$　　d) $\frac{\partial^2 T}{\partial x^2}$

[GATE – 2005]

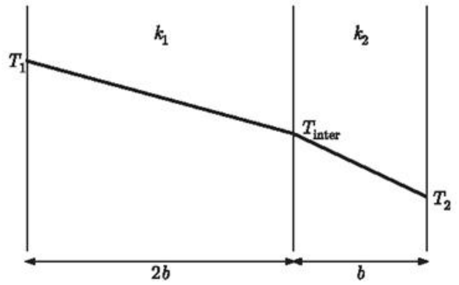
Q.7 Heat flows through a composite slab, as shown below. The depth of the slab is 1 m. The k values are in W/mK. The overall thermal resistance in K/W is



a) 17.2　　b) 21.9

c) 28.6 d) 39.2
[GATE – 2005]

Q.8 In a composite slab, the temperature at the interface (T_{inter}) between two materials is equal to the average of the temperature at the two ends. Assuming steady one-dimensional heat conduction, which of the following statements is true about the respective thermal conductivities?



a) $2k_1 = k_2$ b) $k_1 = k_2$
c) $2k_1 = 3k_2$ d) $k_1 = 2k_2$
[GATE – 2006]

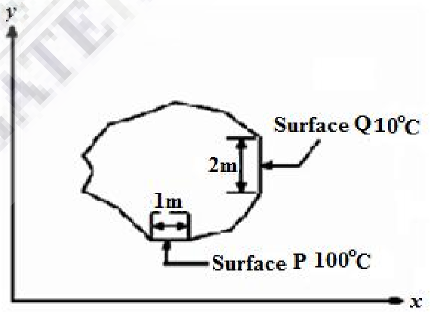
Q.9 With an increase in the thickness of insulation around a circular pipe, heat loss to surrounding due to
a) convection increase, while that the due to conduction decreases
b) convection decrease, while that due to conduction increases
c) convection & conduction decreases
d) convection & conduction increases
[GATE – 2006]

Common Data for Q.10 and Q.11
Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m^3. The left and right faces are kept at constant temperatures of 160°C and 120°C respectively. The plate has a constant thermal conductivity of 200 W/mK.

Q.10 The location of maximum temperature within the plate from its left face is
a) 15 mm b) 10 mm
c) 5 mm d) 0 mm
[GATE – 2007]

Q.11 The maximum temperature within the plate in °C is
a) 160 b) 165
c) 200 d) 250
[GATE – 2007]

Q.12 Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperature gradients over surfaces P and Q can be considered to be uniform. The temperature gradient $\frac{\partial T}{\partial x}$ at surface Q is equal to 10 K/m. Surfaces P and Q are maintained at constant temperature as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of 0.1 W/mK. The values of $\frac{\partial T}{\partial x}$ & $\frac{\partial T}{\partial y}$ at surface P are



a) $\frac{\partial T}{\partial x} = 20K/m, \frac{\partial T}{\partial y} = 0K/m$
b) $\frac{\partial T}{\partial x} = 0K/m, \frac{\partial T}{\partial y} = 10K/m$
c) $\frac{\partial T}{\partial x} = 10K/m, \frac{\partial T}{\partial y} = 10K/m$
d) $\frac{\partial T}{\partial x} = 0K/m, \frac{\partial T}{\partial y}$ = 20K/m
[GATE – 2008]

Q.13 A coolant fluid at 30°C flows over a heated flat plate maintained at constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate may be approximated as T

EXPLANATIONS

Q.1 (c)

Q.2 (c)

Given for water: $T_w = 48°C$,
$k_w = 0.6$ W/mK
And for glass, $T_g = 40°C$, $k_g = 1.2$ W/mK
Spatial gradient in water:

$$\left(\frac{dT}{dy}\right)_w = 1\times10^4 \text{ K/m}$$

At water-glass interface:

$$q = k_w\left(\frac{dT}{dy}\right)_w = K_g\left(\frac{dT}{dy}\right)_g$$

$$\left(\frac{dT}{dy}\right)_g = \frac{K_W}{K_g}\left(\frac{dT}{dy}\right)_w = \frac{0.6}{1.2}\times10^4$$

$$\therefore \left(\frac{dT}{dy}\right)_g = 0.5\times10^4$$

Q.3 (d)

At water-glass interface

$$hA\left[T_w - T_g\right] = K_w A\left(\frac{dT}{dy}\right)_\omega$$

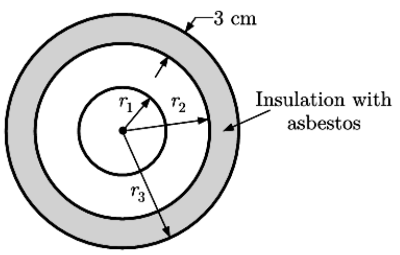
$$\therefore\ h = \frac{K_w\left(\frac{dT}{dy}\right)_w}{(T_w - T_g)}$$

$$= \frac{0.6\times10^4}{(48-40)}$$

$$h = 750 \text{W/m}^2\text{K}$$

Q.4 (b)

Q.5 (c)



Given: $r_1 = \frac{d_1}{2} = 1 \text{ cm}, r_2 = \frac{5}{2} = 2.5 \text{ cm}$,
Radius of asbestos surface,

$$r_3 = \frac{d_2}{2} + 3 = 5.5 \text{ cm},$$

$k_s = 19$ W/mK, $k_a = 0.2$ W/mK,
And $T_1 - T_2 = 600°C$
Equivalent thermal resistance

$$R = \frac{1}{2\pi k_s}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_a}\ln\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\times\pi\times19}\ln\left(\frac{2.5}{1}\right) + \frac{1}{2\times\pi\times0.2}\ln\left(\frac{5.5}{2.5}\right)$$

= 0.635
Heat transfer per unit length.

$$Q = \frac{T_1 - T_2}{R} = \frac{600}{0.635}$$

$= 944.88 \simeq 944.72$ W/m

Q.6 (d)

The general heat equation in Cartesian co-ordinates,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1\partial T}{\alpha\partial t}$$

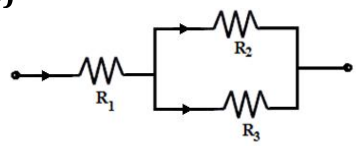
For one dimensional without heat generation the equation reduces to

$$\frac{\partial^2 T}{\partial x^2} = \frac{1\partial T}{\alpha\partial t}$$

For constant properties of medium,

$$\frac{\partial^2 T}{\partial x^2} \propto \frac{\partial T}{\partial t}$$

Q.7 (c)



$$R_1 = \frac{0.5}{0.02\times1} = 25$$

$$R_2 = \frac{0.25}{0.1\times0.5} = 5$$

$$R_3 = \frac{0.25}{0.04\times0.5} = 12.5$$

$$R_{pe} = \frac{R_2 R_3}{R_2 + R_3} = \frac{5\times12.5}{5+12.5}$$
$$= 3.571$$
$$R_{total} = R_1 + R_{pe}$$
$$= 25 + 3.571$$
$$= 28.6\ K/w$$

Q.8 (d)

Given: $T_{inter} = \frac{T_1 + T_2}{2}$

Here Q will be same

$$Q = -\frac{k_1 A_1 (T_1 - T_{inter})}{2b} = \frac{k_2 A_2 (T_{inter} - T_2)}{b}$$

$$Q = \frac{k_1\left[T_1 - \left(\frac{T_1+T_2}{2}\right)\right]}{2b}$$

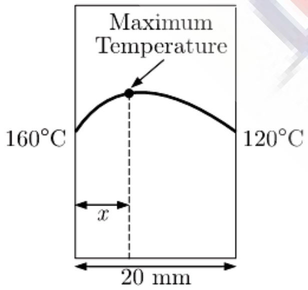
$$= \frac{k_2\left[\left(\frac{T_1+T_2}{2}\right) - T_2\right]}{b}$$

$$\Rightarrow \frac{k_1}{2b} = \frac{k_2}{b}$$

$$k_1 = 2k_2$$

Q.9 (a)

Q.10 (c)



For 1-D steady flow with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{-\dot{q}}{k}$$

Or, $\frac{\partial T}{\partial x} = \frac{-\dot{q}x}{k} + C_1$

Or, $T = \frac{-\dot{q}x^2}{2k} + C_1 x + C_2$ ----- (1)

At x = 0, T = 160°C

$\Rightarrow \quad C_2 = 160$

At x = 0.02 m, T = 120°C

∴ (1) ⇒

$$120 = \frac{-80\times10^6\times0.02^2}{2\times200} + C_1(0.02) + 160$$

$$120 = -80 + C_1(0.02)$$

∴ $C_1 = 2000$

∴ (1) ⇒ $T = \frac{-\dot{q}x^2}{2k} + 2000x + 160$

For maximum temperature: $\frac{dT}{dx} = 0$

$$\Rightarrow \frac{dT}{dx} = \frac{-\dot{q}(2x)}{2k} + 2000 = 0$$

$$\Rightarrow \frac{80\times10^6\times x}{200} = 2000$$

$$x = 5\times10^{-3}\,m$$

∴ x = 5 mm

Q.11 (b)

From the previous question,

$$T = \frac{-qx^2}{2k} + 2000x + 160$$

The temperature is maximum at x = 0.005 m

$$T = \frac{-80\times10^6\times0.005^2}{2\times200} + 2000\times0.005 + 160$$

T = 165°C

Q.12 (d)

Given:

$$\left(\frac{\partial T}{\partial x}\right)_Q = 10\frac{K}{m},\ k = 0.1 W/mK$$

Direction of heat flow is always normal to surface of constant temperature.

So, for surface P,

$$\frac{\partial T}{\partial x} = 0$$

From the law of energy conservation

Heat rate at P = Heat rate at Q

$$-0.1\times1\times\left(\frac{\partial T}{\partial y}\right)_p = -0.1\times2\times\left(\frac{\partial T}{\partial x}\right)_Q$$

(Considering the width to be 1)

EXPLANATIONS

Q.1 (c)

Q.2 (c)
For radiation the magnitude of temperature difference should be large enough. Convection & conduction is also predominant in boiler furnace.

Q.3 (d)
Air, K = 0.024 W/mK
Lead, K = 35 W/mK
Aluminum, K = 234 W/mK
Silver, K = 420 W/mK.

Q.4 (a)
Mica, Bakelite and Fiber glass are insulator material and carbon in the form of diamond is a good conductor of heat but bad conductor of electricity

Q.5 (c)
K_{water}= 0.556 W/m^{0}C
K_{ice} = 2.22 W/m^{0}C
K_{steam} = 0.0206 W/m^{0}C

Q.6 (c)
More heat is lost by forced convection as the value of heat transfer coefficient is large because it depends on fluid properties, velocity of flow and surface condition.

Q.7 (c)
As the radius of insulation around a heated cable gradually increases from zero to critical radius, heat transfer increases and when the radius of insulation exceeds form the critical radius of insulation heat transfer decreases.

Q.8 (b)

Q.9 (b)

Since area is crucial in any mode of heat transfer hence by increasing effective surface area heat transfer rate can be enhanced.

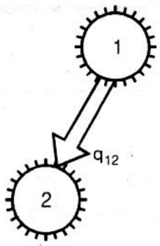
Q.10 (a)
Up to critical thickness heat transfer increases and after critical thickness there will be reduction in heat transfer, when more insulation is provided.

Q.11 (d)

Q.12 (a)

Q.13 (b)

Q.14 (c)
View factor represent the fraction of the energy emitted by one surface and received by the other.



$F_{12} = \frac{q_{12}}{q_1}$

Where
q_1 = Total energy emitted by body 1
q_{12} = Energy emitted by (1) and received by (2)

Q.15 (b)

Q.16 (c)

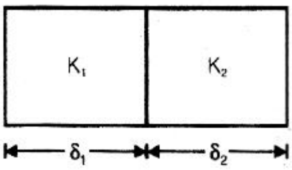
$$\frac{T - T_\infty}{T_i - T_\infty} = e^{\frac{-hA}{\rho Vc}t}$$

Hence temperature variation will be exponential.

Q.17 (c)

Q.18 (d)

Q.19 (c)



$R_{eq} = R_1 + R_2$

$$\frac{\delta_1+\delta_2}{k_{eq}A} = \frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A}$$

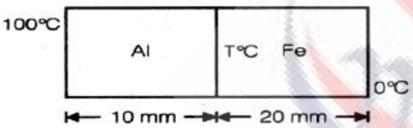
$$\frac{\delta_1+\delta_2}{k_{eq}} = \frac{\delta_1}{k_1} + \frac{\delta_2}{k_2}$$

$$\frac{\delta_1+\delta_2}{k_{eq}} = \frac{\delta_1 k_2 + k_1\delta_2}{k_1 k_2}$$

$$k_{eq} = \frac{k_1 k_2(\delta_1+\delta_2)}{k_1\delta_2 + k_2\delta_1}$$

Q.20 (d)

$$\frac{K_{Al}}{K_{fe}} = 3$$



$$Q_{Al} = \frac{K_{Al}A(100-T)}{10}$$

$$Q_{Fe} = \frac{K_{Fe}\times A(T-0)}{20}$$

$$\frac{K_{Al}(100-T)}{10} = \frac{K_{Fe}(T)}{20}$$

$$\frac{3K_{Fe}(100-T)}{10} = \frac{K_{Fe}(T)}{20}$$

$600 - 6T = T$

$600 = 7T$

$$\Rightarrow T = \frac{600}{7} = 85.7^{o}C$$

Q.21 (a)

Thermal resistance $= \dfrac{L}{KA}$

$$= \frac{0.1}{5\times 200} = 10^{-4}\,k/W$$

Q.22 (a)

Q.23 (c)

$$\frac{Q}{A} = \frac{K(T_1-T_2)}{L}$$

$$25\times 10^3 = \frac{50\times(100-T_2)}{0.1}$$

$T_2 = 50℃$

Q.24 (b)

Since $\Delta T \propto \dfrac{1}{K}$

Q.25 (d)

Q.26 (d)

Q.27 (c)

For sphere

$A_{gm} = \sqrt{A_1 A_2}$

$= \sqrt{2\times 8} = 4\ m^2$

Q.28 (d)

$$q = \frac{T_1-T_2}{R_{total}}$$

$$= \frac{(1840-340)}{\frac{0.3}{0.6\times 1}+\frac{0.2}{0.4\times 1}+\frac{0.1}{0.1\times 1}} = 750\ W$$

Q.29 (c)

Q.30 (a)

$$(Pr)^{\frac{1}{3}} = \frac{\delta}{\delta_1}$$

Since Pr < 1

$\therefore \delta_1 > \delta$

Q.31 (a)

$$Nu = \frac{hD}{K}$$