



# **ELECTROMAGNETIC THEORY**

**For  
ELECTRICAL ENGINEERING  
ELECTRONICS & COMMUNICATION ENGINEERING**



# ELECTROMAGNETIC THEORY

## SYLLABUS

Elements of vector calculus: divergence and curl; Gauss' and Stokes' theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

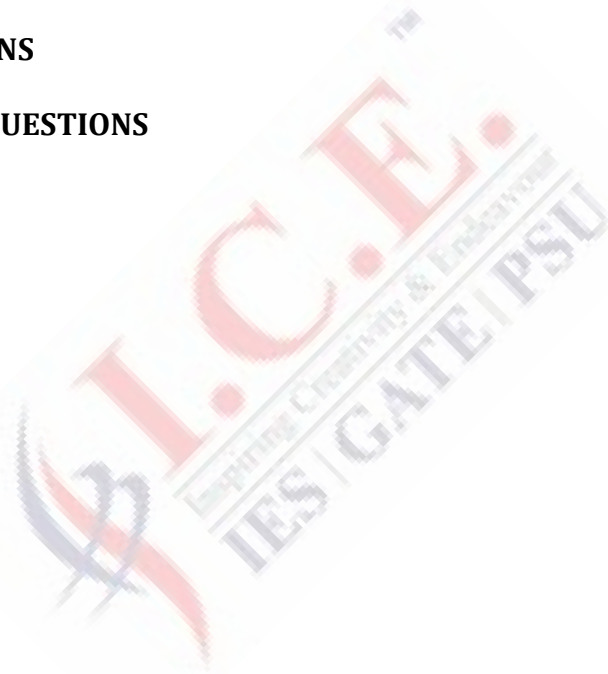
## ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	2	7	16
2004	2	6	14
2005	2	6	14
2006	2	8	18
2007	2	7	16
2008	2	5	12
2009	2	3	8
2010	3	2	7
2011	4	3	10
2012	4	5	14
2013	1	2	5
2014 Set-1	2	3	8
2014 Set-2	2	4	10
2014 Set-3	2	3	8
2014 Set-4	4	3	10
2015 Set-1	2	4	10
2015 Set-2	2	3	8
2015 Set-3	2	4	10
2016 Set-1	2	4	10
2016 Set-2	3	4	11
2016 Set-3	2	4	10
2017 Set-1	1	3	7
2017 Set-2	2	3	8

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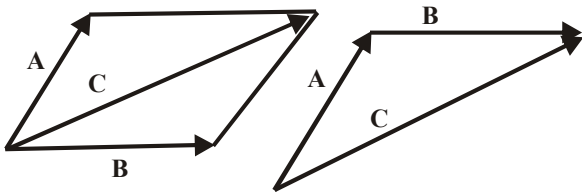
# 1

## VECTORS & COORDINATE SYSTEMS

### 1.1 VECTORS

A vector  $A$  has both magnitude and direction. The magnitude of  $A$  is a scalar written as  $A$  or  $|A|$ . A unit vector  $a_A$  along  $A$  is defined as a vector whose magnitude is unity and its direction is along  $A$ , that is,

$$a_A = \frac{A}{|A|} \quad a_A = \frac{A_x a_x + A_y a_y + A_z a_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$



#### 1.1.1 VECTOR ADDITION & SUBTRACTION

Two vectors  $A$  and  $B$  can be added together to give another vector  $C$ ; that is,  
 $C = A + B$

Law	Addition	Multiplication
Commutative	$A + B = B + A$	$kA = Ak$
Associative	$A + (B + C) = (A + B) + C$	$k(\ell A) = (k\ell)A$
Distributive	$k(A+B) = kA + kB$	

The position vector  $r_p$  (or **radius vector**) of point  $P$  is as the directed from the origin  $O$  to  $P$ ; i.e.,

$$r_p = OP = Xa_x + Ya_y + Za_z$$

The **distance vector** is the displacement from one point to another.

$$r_{PQ} = r_Q - r_P$$

$$= (x_Q - x_P)a_x + (y_Q - y_P)a_y + (z_Q - z_P)a_z$$

#### 1.1.2 VECTOR MULTIPLICATION

- Scalar (or dot) product:  $A \cdot B$
- Vector (or cross) product:  $A \times B$
- Scalar triple product:  $A \cdot (B \times C)$
- Vector triple product:  $A \times (B \times C)$

#### 1. Dot Product

The **dot product** of two vector  $A$  and  $B$ , written as  $A \cdot B$ , is defined geometrically

as the product of the magnitudes of  $A$  and  $B$  and the cosine of the angle between them.

$$A \cdot B = AB \cos \theta_{AB}$$

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

#### 2. Cross Product

$$A \times B = AB \sin \theta_{AB} a_n \quad A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Note that the cross product has the following basic properties:

- It is not commutative:  
 $A \times B \neq B \times A$   
 It is anti-commutative:  
 $A \times B = -B \times A$
- It is not associative:  
 $A \times (B \times C) \neq (A \times B) \times C$
- It is distributive:  
 $A \times (B + C) = A \times B + A \times C$

#### 3. Scalar Triple Product

Given three vectors  $A$ ,  $B$ , and  $C$ , we define the scalar triple product as  
 $A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$

$$A \cdot (B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

#### 4. Vector Triple Product.

$$A \times (B \times C) = B(C \cdot A) - C(A \cdot B)$$

#### 1.1.3 COMPONENTS OF A VECTOR

The vector component  $A_B$  of  $A$  along  $B$  is simply the scalar component

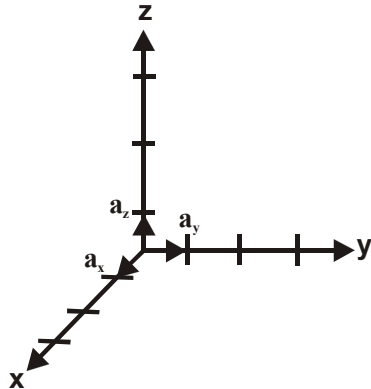
$$A_B = A \cos \theta = \frac{A \cdot B}{|B|}$$

### 1.2 COORDINATE SYSTEMS AND TRANSFORMATION

An **orthogonal system** is one in which the coordinates are mutually perpendicular.

**1.2.1 CARTESIAN COORDINATES (X, Y, Z)**

A point P can be represented as (x, y, z) as in Figure



The ranges of the coordinate variable x, y, and z are

$$-\infty < x < \infty$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$

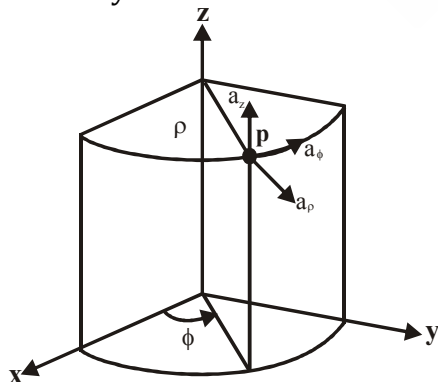
A vector A in **Cartesian** (otherwise known as **rectangular**) coordinates can be written as

$$(A_x, A_y, A_z) \text{ or } A = A_x a_x + A_y a_y + A_z a_z$$

Where,  $a_x$ ,  $a_y$ , and  $a_z$  are unit vectors along the x, y and z directions as shown in Figure.

**1.2.2 CIRCULAR CYLINDRICAL COORDINATES ( $\rho, \phi, z$ )**

Azimuthal angle, is measured from the x-axis in the xy-plane; and z is the same as in the Cartesian system.



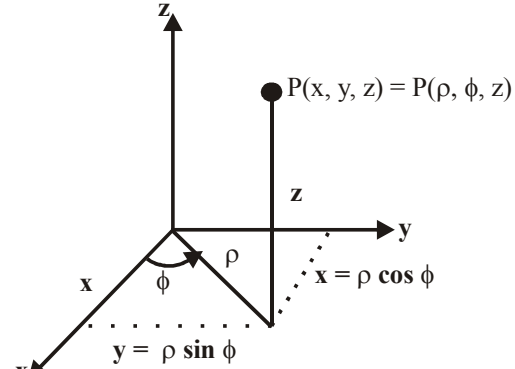
The ranges of the variables are

$$0 \leq \rho < \infty$$

$$0 \leq \phi < 2\pi$$

$$-\infty < z < \infty$$

A vector A in cylindrical coordinates can be written as



$$(A_\rho, A_\phi, A_z) \text{ or } A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

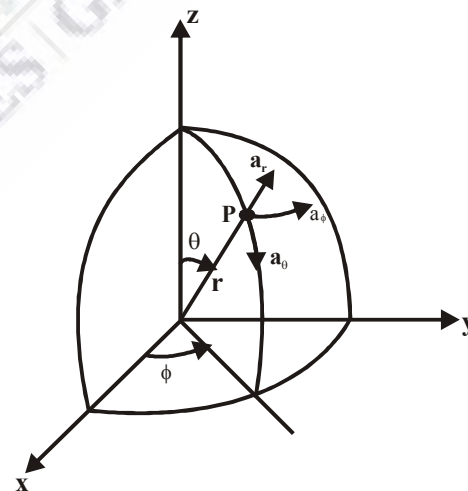
$$\rho = \sqrt{x^2 + y^2}, \phi = \tan^{-1} \frac{y}{x}, z = z \text{ or}$$

$$x = \rho \cos \phi, y = \rho \sin \phi, z = z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \text{ or}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

**1.2.3 SPHERICAL COORDINATES ( $r, \theta, \phi$ )**



**Range:**

$$0 \leq r < \infty, 0 \leq \theta \leq \pi \text{ \& } 0 \leq \phi < 2\pi$$

$$(A_r, A_\theta, A_\phi) \text{ or } A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

$$|A| = (A_r^2 + A_\theta^2 + A_\phi^2)^{1/2}$$

$$r = \sqrt{x^2 + y^2 + z^2}, \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z},$$

$$\phi = \tan^{-1} \frac{y}{x} \text{ or}$$

$$x = r \sin \theta \cos \varphi, y = r \sin \theta \sin \varphi, z = r \cos \theta$$

### 1.2.4 CONSTANT-COORDINATE SURFACES

- X = Constant
- Y = Constant
- Z = Constant
- $\rho$  = Constant
- $\varphi$  = Constant
- Z = Constant
- r = Constant
- $\theta$  = Constant

### 1.3 DIFFERENTIAL LENGTH, AREA, & VOLUME

#### A. Cartesian Coordinates

- 1) Differential displacement is given by  
 $dx a_x + dy a_y + dz a_z$
- 2) Different normal area is given by  
 $dS = dy dz a_x$  or  
 $= dx dz a_y$  or  
 $= dz dy a_z$
- 3) Different volume is given by  
 $V = dx dy dz$

#### B. Cylindrical Coordinates

- 1) Different displacement is given by  
 $dl = d\rho a_\rho + \rho d\varphi a_\varphi + dz a_z$
- 2) Different normal area is given by  
 $dS = \rho d\varphi dz a_\rho$  or  
 $= d\rho dz a_\varphi$  or  
 $= \rho d\varphi d\rho a_z$
- 3) Differential volume is given by  
 $dV = \rho d\rho d\varphi dz$

#### C. Spherical Coordinates

- 1) The different displacement is  
 $dl = dr a_r + r d\theta a_\theta + r \sin \theta d\varphi a_\varphi$
- 2) The differential normal area is  
 $dS = r^2 \sin \theta d\theta d\varphi a_r$  or  
 $= r \sin \theta dr d\varphi a_\theta$  or  
 $= r dr d\theta a_\varphi$
- 3) The differential volume is  
 $dV = r^2 \sin \theta dr d\theta d\varphi$

The **line integral**  $\int_L A \cdot dl$  is the integral of the tangential component of A along curve L.

Given a vector field A, continuous in a region containing the smooth surface S, we define the surface Integral or the flux of A through S as

$$\psi = \int_S A \cdot dS$$

### 1.4 DEL OPERATOR

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

$$\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial}{\partial \varphi} a_\varphi + \frac{\partial}{\partial z} a_z$$

$$\nabla = \frac{\partial}{\partial r} a_r + \frac{1}{r} \frac{\partial}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} a_\varphi$$

#### 1.4.1 APPLICATION OF DEL OPERATOR

- The gradient of a scalar V, written as  $\nabla V$
- The divergence of a vector A, written as  $\nabla \cdot A$
- The curl of a vector A, written as  $\nabla \times A$
- The Laplacian of a scalar V, written  $\nabla^2 V$

#### 1.4.2 GRADIENT OF A SCALAR

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \varphi} a_\varphi + \frac{\partial V}{\partial z} a_z$$

$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \varphi} a_\varphi$$

#### 1.4.3 DIVERGENCE OF A VECTOR

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

$$\text{div} A = \nabla \cdot A = \lim_{\Delta v \rightarrow 0} \frac{\oint_S A \cdot dS}{\Delta v}$$



$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

### 1.4.4 DIVERGENCE THEOREM

The **divergence theorem** states that the total outward flux of a vector field  $\mathbf{A}$  through the closed surface  $S$  is the same as the volume integral of the divergence of  $\mathbf{A}$ .

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{A} \, dv$$

### 1.4.5 CURL OF A VECTOR

The **curl** of  $\mathbf{A}$  is an axial (or rotational) vector whose magnitude is the maximum circulation of  $\mathbf{A}$  per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented so as to make the circulation maximum. i. e.

$$\text{Curl } \mathbf{A} = \nabla \times \mathbf{A} = \left( \lim_{\Delta S \rightarrow 0} \frac{\oint_L \mathbf{A} \cdot d\mathbf{l}}{\Delta S} \right) \mathbf{a}_{n_{\max}}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho} \begin{vmatrix} \mathbf{a}_\rho & \rho \mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \mathbf{a}_r & r \mathbf{a}_\theta & r \sin \theta \mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

### 1.4.6 STOKES'S THEOREM

**Stokes's theorem** states that the circulation of a vector field  $\mathbf{A}$  around a (closed) path  $L$  is equal to the surface integral of the curl of  $\mathbf{A}$  over the open surface  $S$  bounded by  $L$  provided that  $\mathbf{A}$  and  $\nabla \times \mathbf{A}$  are continuous on  $S$ .

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

### 1.4.7 LAPLACIAN OF A SCALAR

The Laplacian of a scalar field  $V$ , written as

$\nabla^2 V$ , is the divergence of the gradient of  $V$ .

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

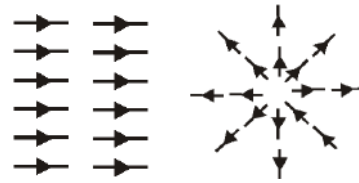
$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

### 1.5 CLASSIFICATION OF VECTOR FIELDS

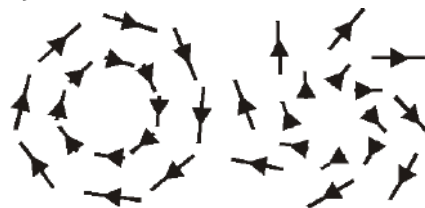
a)  $\nabla \cdot \mathbf{A} = 0, \nabla \times \mathbf{A} = 0$

b)  $\nabla \cdot \mathbf{A} \neq 0, \nabla \times \mathbf{A} = 0$



c)  $\nabla \cdot \mathbf{A} = 0, \nabla \times \mathbf{A} \neq 0$

d)  $\nabla \cdot \mathbf{A} \neq 0, \nabla \times \mathbf{A} \neq 0$



# GATE QUESTIONS

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**Q.1** The electric field on the surface of a perfect conductor is 2 V/m. The conductor is immersed in water with  $\epsilon = 80\epsilon_0$ . The surface charge density on the conductor is

- a)  $0\text{C}/\text{m}^2$   
 b)  $2\text{C}/\text{m}^2$   
 c)  $1.8 \times 10^{-11}\text{C}/\text{m}^2$   
 d)  $1.41 \times 10^{-9}\text{C}/\text{m}^2$   
 ( $\epsilon = 10^9$ ) /  $(36\pi\text{F}/\text{m})$

[GATE - 2002]

**Q.2** if the electric field intensity is given by  $E = (xu_x + yu_y + zu_z)\text{volt}/\text{m}$  the potential difference between X (2, 0, 0) and Y (1, 2, 3) is

- a) + 1 volt  
 b) - 1 volt  
 c) + 5 volt  
 d) + 6 volt

[GATE - 2003]

**Q.3** The unit  $\nabla \times H$  is

- a) Ampere  
 b) Ampere/meter  
 c) Ampere/meter<sup>2</sup>  
 d) Ampere-meter

[GATE - 2003]

**Q.4** A parallel plate air - filled capacitor has plate area of  $10^{-4}\text{m}^2$  and plate separation of  $10^{-3}\text{m}$ . It is connected to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is

$$(\epsilon_0 = 1/36\pi \times 10^{-9}\text{F}/\text{m})$$

- a) 10 mA  
 b) 100 mA  
 c) 10 A  
 d) 1.59 mA

[GATE - 2004]

**Q.5** If C is a closed curve enclosing a surface S, then the magnetic field intensity  $\vec{H}$ , the current density  $\vec{J}$  and the electric flux density  $\vec{D}$  are related by

$$\text{a) } \iint_S \vec{H} \cdot d\vec{s} = \oint_C \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{l}$$

$$\text{b) } \int_C \vec{H} \cdot d\vec{l} = \oiint_S \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$$

$$\text{c) } \oiint_S \vec{H} \cdot d\vec{s} = \int_C \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{l}$$

$$\text{d) } \oint_C \vec{H} \cdot d\vec{l} = \iint_S \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$$

[GATE - 2007]

**Q.6** For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of two of Maxwell's equations?

- a)  $\nabla \cdot E = 0$   
 b)  $\nabla \cdot E = 0 \quad \nabla \times B = 0 \quad \nabla \cdot B = 0$   
 c)  $\nabla \times E = 0$   
 d)  $\nabla \times E = 0 \quad \nabla \times B = 0 \quad \nabla \cdot B = 0$

[GATE - 2008]

**Q.7** A magnetic field in air is measured to be  $\vec{B} = B_0 \left( \frac{x}{x^2 + y^2} \hat{y} - \frac{y}{x^2 + y^2} \hat{x} \right)$  what current distribution leads to this field?

[Hint : the algebra is trivial in cylindrical coordinates.]

$$\text{a) } \vec{j} = -\frac{B_0 Z}{\mu_0} \left( \frac{1}{x^2 + y^2} \right), r \neq 0$$

$$\text{b) } \vec{j} = -\frac{B_0 Z}{\mu_0} \left( \frac{2}{x^2 + y^2} \right), r \neq 0$$

$$\text{c) } \vec{j} = 0, r \neq 0$$

$$\text{d) } \vec{j} = -\frac{B_0 Z}{\mu_0} \left( \frac{1}{x^2 + y^2} \right), r \neq 0$$

[GATE - 2009]

# EXPLANATIONS

**Q.1 (d)**

$$D = \rho_s = \epsilon E = 80 \cdot \epsilon \cdot 2$$

**Q.2 (c)**

$$\begin{aligned} V &= -\int E dl \\ &= -\left[ \int_1^2 x dx \vec{u}_x + \int_2^0 y dy \vec{u}_y + \int_3^0 z dz \vec{u}_z \right] \\ &= -\left[ \frac{X^2}{2} \Big|_1^2 + \frac{y^2}{2} \Big|_2^0 + \frac{z^2}{2} \Big|_3^0 \right] \\ &= -\frac{1}{2} [2^2 - 1^2 + 0^2 - 2^2 + 0^2 - 3^2] \\ &= -\frac{1}{2} \times -10 = 5V \end{aligned}$$

**Q.3 (c)**

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

J is current density = A/m<sup>2</sup>

**Q.4 (a)**

Displacement current

$$\begin{aligned} I_d &= A J_d \\ &= A \frac{\partial D}{\partial t} = A \epsilon \frac{\partial E}{\partial t} \\ |I_d| &= |A \epsilon \omega E| \\ &= A \omega \epsilon \frac{V}{d} \end{aligned}$$

After putting values we get

$$I_d = 10 \text{ mA}$$

**Q.5 (d)**

**Q.6 (d)**

**Q.7 (c)**

$$\vec{B} = B_0 \left( \frac{x}{x^2 + y^2} \hat{a}_y - \frac{y}{x^2 + y^2} \hat{a}_x \right)$$

Convert to cylindrical coordinates and put

$$x = r \cos \phi$$

$$\hat{a}_x = \cos \phi \hat{a}_r - \sin \phi \hat{a}_\phi$$

$$y = r \sin \phi$$

$$\hat{a}_y = \sin \phi \hat{a}_r + \cos \phi \hat{a}_\phi$$

Putting the values

$$\vec{B} = B_0 \hat{a}_\phi$$

$$\Rightarrow \vec{H} = \frac{B_0}{\mu_0} \hat{a}_\phi = \text{const} \tan t$$

$$\vec{J} = \nabla \times \vec{H} = \nabla \times [\text{const} \tan t] = 0$$

**Q.8 (b)**

This represents stoke's theorem

$$\oint_C \vec{A} \cdot d\vec{t} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{s} = \iint_S \vec{V} \cdot d\vec{s}$$

**Q.9 (d)**

**Q.10 (d)**

$$\begin{aligned} \oiint_S 5\vec{r} \cdot \hat{n} ds &= \iiint_V \nabla \cdot 5\vec{r} dv \\ &= 5 \iiint_V \nabla \cdot \vec{r} dv = 5 \times 3 \\ &= 15 \text{ volt} \end{aligned}$$

**Q.11 (d)**

Given

$$\vec{E} = E_\rho e^{j(\omega t - 280\pi y)} \hat{u}_z \text{ V/m}$$

$$\vec{H} = 3e^{j(\omega t - 280\pi y)} \hat{u}_x \text{ A/m}$$

From given expression we conclude that

$$\beta = 280\pi = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{1}{140} \text{ meter}$$

$$v = f \lambda$$

$$= 14 \times 10^9 \times \frac{1}{140} \text{ m/sec}$$

$$v = 1 \times 10^8 \text{ m/sec}$$

# ASSIGNMENT QUESTIONS

**Q.1** Which of the following statement regarding electric flux is true?

1. Electric flux begins on positive charges and terminates on negative charges.
2. Flux is in the same direction as the electric field  $\vec{E}$
3. Flux density is proportional to the magnitude of  $\vec{E}$
4. In the SI system of units, total flux emanating from a charge of  $Q(C)$  is  $Q(C)$ .

A single line will emanate from 1 C of charge.

- a) 1 only                      b) 1 & 2 only  
c) 1, 2 & 3 only              d) 1, 2, 3 & 4

**Q.2** Two concentric spherical shells carry equal and opposite uniformly distributed charges over their surfaces as shown in the figure. The electric field on the surface of the inner shell will be

- a) zero                      b)  $\frac{Q}{4\pi\epsilon_0 R^2}$   
c)  $\frac{Q}{8\pi\epsilon_0 R^2}$                   d)  $\frac{Q}{16\pi\epsilon_0 R^2}$

**Q.3** Joule/Coulomb is the unit of

- a) Electric field potential  
b) Electric flux density  
c) Charge  
d) None of the above

**Q.4** The electric potential due to an electric dipole of length  $L$  at a point distance  $r$  away from it will be doubled if the

- a) length  $L$  of the dipole is doubled  
b)  $r$  is doubled  
c)  $r$  is halved  
d)  $L$  is halved

**Q.5** The force between two charged particles is given by,

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}, \text{ where the symbols}$$

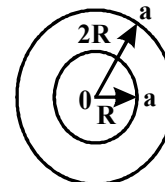
have their usual meanings. The dimensions of  $\epsilon_0$  in free space in SI system are:

- a)  $M^{-1} L^{-3} T^6 A^4$               b)  $M^{-1} L^{-3} T^4 A^2$   
c)  $ML^{-3} T^4 A^3$                   d)  $M^{-1} L^{-3} T^2$

**Q.6** The energy spent in moving a charge of 10 coulomb from one point 'a' to another point 'b' is 50 joules. The potential difference between points 'a' and 'b' is

- a) 2 volts                      b) 5 volts  
c) 10 volts                      d) 100 volts

**Q.7** P is a point at a large distance from the centre O of a short dipole formed by two point charges all lying on a horizontal plane. If  $\theta$  is the angle between OP and the dipole axis, then  $\theta$ , component of the E-field at P is



- a) given by  $\sin \theta$     b) given by  $\cos \theta$   
c) given by  $\tan \theta$     d) independent of

$\theta$

**Q.8** Two equal positive point charges are placed along X-axis at  $+X_1$  and  $-X_1$  respectively. The electric field vector at a point P on the positive Y axis will be directed

- a) in the +X direction  
b) in the -X direction  
c) in the +Y direction  
d) in the -Y direction

**Q.9)** Two concentric spherical conducting shells are held at two different potentials. Their centre coincides with the origin of a spherical  $(r, \theta)$

# EXPLANATIONS

**Q.1 (c)**

\*Electric flux flows from +ve charge to negative charge in the form of field lines.

\*Direction of flux and field lines is same

\* Flux=charge

\* but only one line will not emanate from 1C of charge

**Q.2 (b)**

The electric field on the surface of inner shell is

$$E = \frac{Q}{4\pi\epsilon_0 R^2}$$

as the inner shell and outer shell is separated by a distance R.

**Q.3 (a)**

$$V = \frac{W}{q}$$

Potential is work done per unit charge

**Q.4 (a)**

Potential due to an electric dipole is

$$V = \frac{Q}{4\pi\epsilon_0} \frac{L\cos\theta}{r^2}$$

**Q.5 (b)**

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$$N = \frac{1}{\epsilon_0} \frac{c^2}{m^2}$$

$$\epsilon_0 = N^{-1} m^{-2} C^2$$

$$= (kgm/s^2)^{-1} m^{-2} (As)^2$$

$$= kg^{-1} m^{-3} S^4 A^2$$

$$[M^{-1} L^{-3} T^4 A^2]$$

**Q.6 (b)**

Potential difference is given by

$$v = \frac{\text{workdone}}{\text{charge}} \text{ or } \frac{\text{Energy}}{\text{charge}}$$

$$= \frac{50}{10} = 5\text{volts}$$

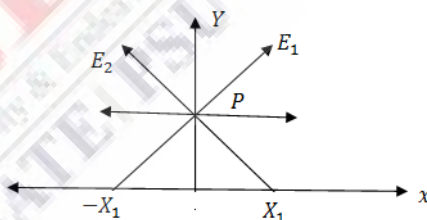
**Q.7 (a)**

$\vec{E}$  due to a dipole is given by

$$\vec{E} = \frac{Qd}{4\pi\epsilon_0 r^3} [2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta] V/m$$

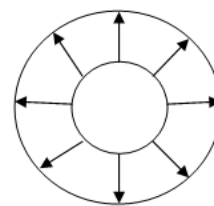
$\theta$  component is given by  $\sin\theta$

**Q.8 (c)**



X-component will cancel each other hence the electric field will be only in +y - direction.

**Q.9 (b)**



Assuming inner shell at higher potential

Electric field will directed from higher potential to lower potential and it varies with distance as  $r^{-2}$ .

**Q.10 (c)**

The correct matching is

$$\text{Faraday's Law: } V = -\frac{d\phi_m}{dt}$$